Further analysis of various equations concerning the Ramanujan Manuscript Book 1. New possible mathematical connections with various topics regarding the Dark Matter, the String Theory, in particular the Supersymmetry Breaking.
XIII

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Abstract

In this research thesis, (part XIII) we analyze further equations concerning the Ramanujan Manuscript Book 1. We obtain new possible mathematical connections with various topics regarding the Dark Matter, the String Theory, in particular the Supersymmetry Breaking.

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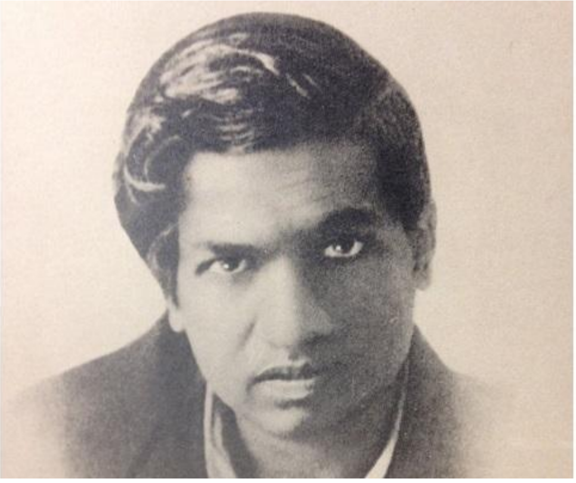
Srinivasa Ramanujan (1887-1920)



https://www.moduscc.it/ramanujan-il-grande-matematico-indiano-13453-131115/

Vesuvius landscape with gorse – Naples

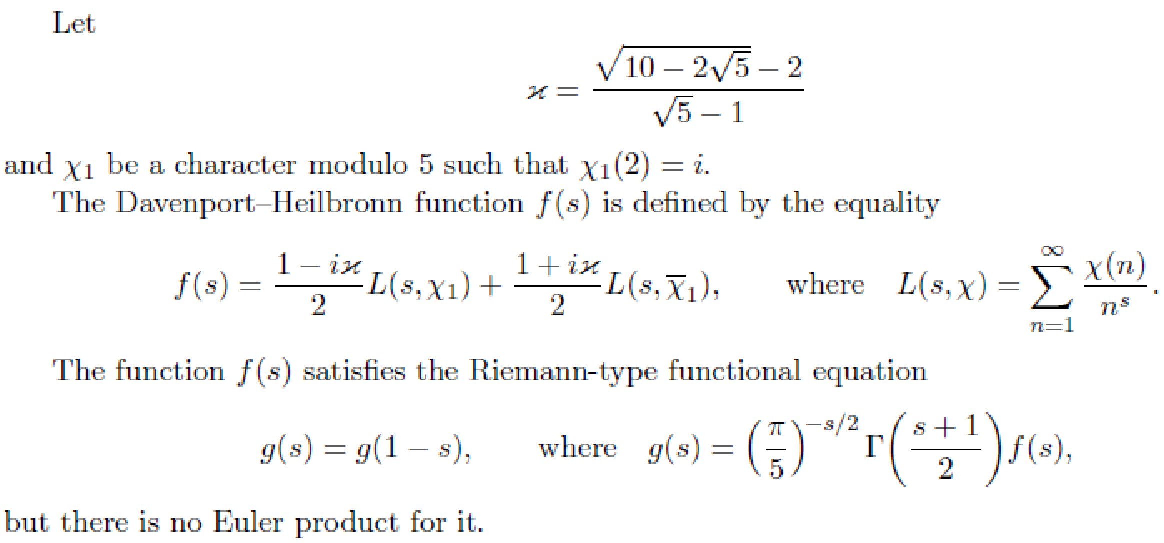
https://www.pinterest.it/pin/95068242114589901/

From:

On the Zeros of the Davenport Heilbronn Function
S. A. Gritsenko - Received May 15, 2016 - ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2017, Vol. 296, pp. 65–87.

A. A. Karatsuba, “On the zeros of arithmetic Dirichlet series without Euler product,” Izv. Ross. Akad. Nauk, Ser. Mat. 57 (5), 3–14 (1993)

We have:



Let

$$\varkappa = \frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

and χ_1 be a character modulo 5 such that $\chi_1(2) = i$.

The Davenport–Heilbronn function $f(s)$ is defined by the equality

$$f(s) = \frac{1-i\varkappa}{2}L(s,\chi_1) + \frac{1+i\varkappa}{2}L(s,\overline{\chi}_1), \qquad \text{where} \quad L(s,\chi) = \sum_{n=1}^{\infty}\frac{\chi(n)}{n^s}.$$

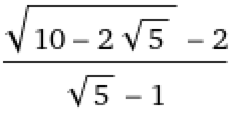
The function $f(s)$ satisfies the Riemann-type functional equation

$$g(s) = g(1-s), \qquad \text{where} \quad g(s) = \left(\frac{\pi}{5}\right)^{-s/2}\Gamma\left(\frac{s+1}{2}\right)f(s),$$

but there is no Euler product for it.

$$(\sqrt{10-2\sqrt{5}}-2)/(\sqrt{5}-1) = \kappa$$

Input:



$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

Decimal approximation:

0.2840790438404122960282918323931261690910880884457375827591626661

…

0.28407904384…. = κ

Alternate forms:

$$\frac{1}{4}\left(\sqrt{10-2\sqrt{5}}-2\sqrt{5}+\sqrt{5\left(10-2\sqrt{5}\right)}-2\right)$$

$$\frac{1}{4}\left(1+\sqrt{5}\right)\left(\sqrt{10-2\sqrt{5}}-2\right)$$

$$\frac{1}{2}\left(-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

Minimal polynomial:

$$x^4+2x^3-6x^2-2x+1$$

Expanded forms:

$$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}-1}-\frac{2}{\sqrt{5}-1}$$

$$\frac{1}{4}\sqrt{10-2\sqrt{5}}+\frac{1}{4}\sqrt{5\left(10-2\sqrt{5}\right)}+\frac{1}{2}\left(-1-\sqrt{5}\right)$$

For ((((√(10-2√5) -2))⁄((√5-1)))) = 8πG;   G = 0.011303146014

Indeed:

((((√(10-2√5) -2))⁄((√5-1))))/(8π)

Input:

$$\frac{\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}{8\pi}$$

Result:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{8\left(\sqrt{5}-1\right)\pi}$$

Decimal approximation:

0.011303146014005214797375012944203574468576031392001780859490966 7…

0.01130314…. = g (gravitational coupling constant)

Property:

$$\frac{-2+\sqrt{10-2\sqrt{5}}}{8\left(-1+\sqrt{5}\right)\pi}$$ is a transcendental number

Alternate forms:

$$\frac{\sqrt{10-2\sqrt{5}}-2\sqrt{5}+\sqrt{5\left(10-2\sqrt{5}\right)}-2}{32\pi}$$

$$-\frac{1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}}{16\pi}$$

$$\frac{-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}}{16\pi}$$

Expanded forms:

$$-\frac{1}{16\pi}-\frac{\sqrt{5}}{16\pi}+\frac{\sqrt{10-2\sqrt{5}}}{32\pi}+\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{32\pi}$$

$$\frac{\sqrt{10-2\sqrt{5}}}{8\left(\sqrt{5}-1\right)\pi}-\frac{1}{4\left(\sqrt{5}-1\right)\pi}$$

Series representations:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)}=\frac{-2+\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}}{8\pi\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)}=\frac{-2+\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}}{8\pi\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)}=\frac{-2+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}}{8\pi\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

We note that:

(((√(10-2√5) -2))/((√5-1)))*((2 i (sqrt(5) - 1) t + sqrt(5) - 1)/(2 (sqrt(2 (5 - sqrt(5))) - 2)))

Input:

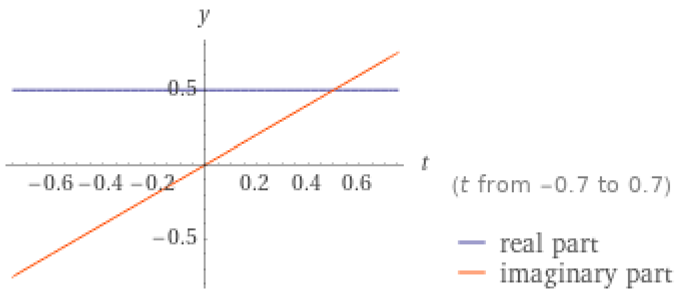
$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\times\frac{2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

i is the imaginary unit

Exact result:

$$\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

Plot:



Alternate form assuming t>0:

$$\frac{i\sqrt{10-2\sqrt{5}}\;t}{\sqrt{2\left(5-\sqrt{5}\right)}-2}-\frac{2\,i\,t}{\sqrt{2\left(5-\sqrt{5}\right)}-2}+$$
$$\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}-\frac{\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}-$$
$$\frac{\sqrt{5}}{\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}+\frac{1}{\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

Alternate forms:

$$\frac{1}{8}\left(1+\sqrt{5}\right)\left(2\,i\sqrt{2\left(3-\sqrt{5}\right)}\;t+\sqrt{5}-1\right)$$

$$\frac{1}{2}\left(1+2\,i\,t\right)$$

$$\frac{1}{2}+i\,t$$

1/2+*it* = *real part of every nontrivial zero of the Riemann zeta function*

Derivative:

$$\frac{d}{dt}\left(\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{\left(\sqrt{5}-1\right)\left(2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)\right)}\right)=i$$

Indefinite integral:

$$\int \frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{\left(\sqrt{5}-1\right)\left(2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)\right)}\,dt=\frac{t}{2}+\frac{i\,t^2}{2}+\text{constant}$$

And again:

(((√(10-2√5) -2))/((2x)))*((2 i (sqrt(5) - 1) t + sqrt(5) - 1)/(2 (sqrt(2 (5 - sqrt(5))) - 2))) = (1/2+it)

Input:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{2\,x}\times\frac{2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}=\frac{1}{2}+i\,t$$

i is the imaginary unit

Exact result:

$$\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}=\frac{1}{2}+i\,t$$

Alternate form assuming t and x are real:

$$\frac{\sqrt{5}-1}{x}=2$$

Alternate form:

$$\frac{\left(\sqrt{5}-1\right)(1+2\,i\,t)}{4\,x}=\frac{1}{2}+i\,t$$

Alternate form assuming t and x are positive:

$$2\,x+1=\sqrt{5}$$

Expanded forms:

$$\frac{i\sqrt{5\left(10-2\sqrt{5}\right)}\;t}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{i\sqrt{10-2\sqrt{5}}\;t}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{i\sqrt{5}\;t}{\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{i\,t}{\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{\sqrt{10-2\sqrt{5}}}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{\sqrt{5}}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}=\frac{1}{2}+i\,t$$

$$\frac{i\sqrt{5}\;t}{2\,x}-\frac{i\,t}{2\,x}+\frac{\sqrt{5}}{4\,x}-\frac{1}{4\,x}=\frac{1}{2}+i\,t$$

Solutions:

$$t=\frac{i}{2}\,,\quad x\neq 0$$

$$x=\frac{\sqrt{5}}{2}-\frac{1}{2}$$

Input:

$$\frac{\sqrt{5}}{2} - \frac{1}{2}$$

Decimal approximation:

0.6180339887498948482045868343656381177203091798057628621354486227
…

0.6180339887…. $= \frac{1}{\phi}$

Solution for the variable x:

$$x = \frac{-2\,i\,\sqrt{5}\,t + 2\,i\,t - \sqrt{5} + 1}{-2 - 4\,i\,t}$$

Implicit derivatives:

$$\frac{\partial x(t)}{\partial t} = \frac{2\left(-1 + \sqrt{5} - 2\,x\right)x}{\left(-1 + \sqrt{5}\right)(-i + 2\,t)}$$

$$\frac{\partial t(x)}{\partial x} = \frac{\left(-1 + \sqrt{5}\right)(-i + 2\,t)}{2\left(-1 + \sqrt{5} - 2\,x\right)x}$$

Dark Energy and Dark Matter

We consider Λ = 1.1056 * 10^{-52} that is the value of the Cosmological Constant and 299792458 , that is the value of the speed of light in *m/s* , the terms of the Einstein equation $E = mc^2$ (E = 1.1056 * 10^{-52} (Λ = Dark Energy)). Thence, we can to obtain the mass *m*:

(1.1056*10^-52)/(299792458)^2

Input interpretation

$$\frac{1.1056\times 10^{-52}}{299\,792\,458^2}$$

Result

$$1.2301459019728805386116738651359598467212878595555184012098\ldots\times 10^{-69}$$

1.2301459019…*10^{-69}

From (**Ramanujan Manuscript Book 1** – *Srinivasa Ramanujan*):

From:

$$\frac{d}{d\beta}\left(\frac{1+\left(\frac{1}{2}\right)^2(1-\beta)+\left(\frac{3}{2\times 4}\right)^2(1-\beta)^2}{1+\left(\frac{1}{2}\right)^2\beta+\left(\frac{3}{2\times 4}\right)^2\beta^2}\right)=\frac{450\left(\beta^2-\beta-8\right)}{\left(9\beta^2+16\beta+64\right)^2}$$

we obtain, for β = 8.7 and 37.6 :

(450 (-8 - 8.7 + 8.7^2))/(64 + 16 8.7 + 9 8.7^2)^2

Input

$$\frac{450\left(-8-8.7+8.7^2\right)}{\left(64+16\times 8.7+9\times 8.7^2\right)^2}$$

Result

0.0339377949190686866502717045312159360154663404711556830674890395

...

0.033937794919….

(450 (-8 - 37.6 + 37.6^2))/(64 + 16 37.6 + 9 37.6^2)^2

Input

$$\frac{450\left(-8-37.6+37.6^2\right)}{\left(64+16\times 37.6+9\times 37.6^2\right)^2}$$

Result

0.0034341920297328709820193684390072521962041927464073680700048781

...

0.00343419202….

From the two previous analyzed expressions:

((((450 (-8 - 8.7 + 8.7^2))/(64 + 16 8.7 + 9 8.7^2)^2 + (450 (-8 - 37.6 + 37.6^2))/(64 + 16 37.6 + 9 37.6^2)^2)))^48

Input

$$\left(\frac{450\left(-8-8.7+8.7^2\right)}{\left(64+16\times 8.7+9\times 8.7^2\right)^2}+\frac{450\left(-8-37.6+37.6^2\right)}{\left(64+16\times 37.6+9\times 37.6^2\right)^2}\right)^{48}$$

Result

$3.0354078186632568216366107605966769350712011813380301580071\ldots \times 10^{-69}$

$3.035407818\ldots*10^{-69}$

And:

1/(([2.6 -25/9 ln(2.6(9*2.6+16)+64)+1/(144*sqrt2)*(((625*tan^-1((9*2.6+8)/(16sqrt2)))))]+[17.6 -25/9 ln(17.6(9*17.6+16)+64)+1/(144*sqrt2)*(((625*tan^-1((9*17.6+8)/(16sqrt2)))))]))^72

Input

$$1\Big/\left(\left(2.6-\frac{25}{9}\log(2.6(9\times 2.6+16)+64)+\frac{1}{144\sqrt{2}}\left(625\tan^{-1}\left(\frac{9\times 2.6+8}{16\sqrt{2}}\right)\right)\right)+\left(17.6-\frac{25}{9}\log(17.6(9\times 17.6+16)+64)+\frac{1}{144\sqrt{2}}\left(625\tan^{-1}\left(\frac{9\times 17.6+8}{16\sqrt{2}}\right)\right)\right)\right)^{72}$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Result

$1.24433\ldots \times 10^{-69}$

(result in radians)

$1.24433\ldots*10^{-69}$

we obtain, after some calculations:

(16 sqrt(2/3) log^(3/2)(2))/(e^(5/2) log^2(3)) ((1/(1+(zeta(2))^2)(3.035407*10^-69 + 1.24433× 10^-69)+1.24433× 10^-69))

Where

$$\frac{16\sqrt{\frac{2}{3}}\log^{3/2}(2)}{e^{5/2}\log^2(3)}$$

= 0.5127284721865…..

Input interpretation

$$\frac{16\sqrt{\frac{2}{3}}\log^{3/2}(2)}{e^{5/2}\log^2(3)}\left(\frac{1}{1+\zeta(2)^2}\left(3.035407\times10^{-69}+1.24433\times10^{-69}\right)+1.24433\times10^{-69}\right)$$

$\log(x)$ is the natural logarithm

$\zeta(s)$ is the Riemann zeta function

Result

$1.23014\ldots\times10^{-69}$

$1.23014\ldots*10^{-69}$

From:

Connecting the Extremes: A Story of Supermassive Black Holes and Ultralight Dark Matter - *Hooman Davoudiasl, Peter B. Denton and Julia Gehrlein* - arXiv:2109.01678v1 [astro-ph.CO] 3 Sep 2021

We have:

In this work, we considered a possible connection between the observations of $\sim 10^9 M_\odot$ supermassive black holes (SMBHs) at redshifts $z \sim 6-7$ and the possibility that dark matter (DM) is an ultralight axion of mass close to $\sim 10^{-20}$ eV. The appearance of SMBHs at such an early epoch poses a puzzle. Apart from being an interesting possibility, such axions may address certain small scale features of DM distribution and may be favored by measurements of ultra faint dwarfs. The connection that we propose is a dark sector confining first order phase transition, characterized by scales of $\mathcal{O}(10 \text{ keV})$, that provides a catalyst for the primordial formation of $\sim 10^9 M_\odot$ SMBHs and endows the ultralight axion with mass, in a fashion similar to the QCD axion.

Here we have assumed that the relation between the size of the BH and the horizon scale is $R \sim \epsilon/H$ and that the temperature at BH formation is given approximately by the energy scale of the FOPT $\mu_a \sim T$ such that the relation between the axion mass and the SMBH mass can be written as

$$m_a = \varepsilon' \frac{M_{\rm P}^3}{f_a M_{BH}} \tag{11}$$

where ε' encompasses deviations from the correspondence $\mu_a \sim T$, and $R \sim 1/H$. In Fig. 1 we show two benchmark points for $f_a \in \{10^{17}, 10^{18}\}$ GeV and ε' between 0.01 and 1. For both benchmark points we find allowed regions which can explain the observed pSMBH population with axion masses not constrained yet. The compatible region for f_a may also be suggested by string theory [16] as mentioned earlier.

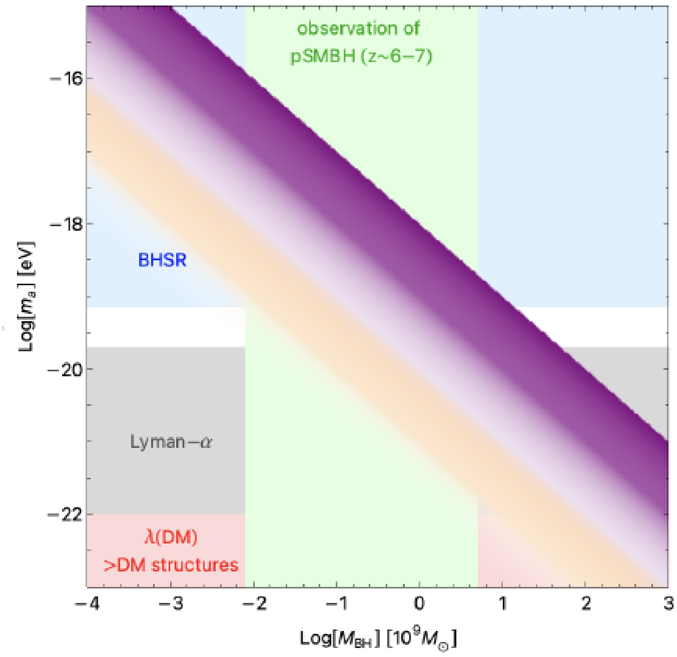


FIG. 1. Summary plot of the constraints and preferred regions of our model in the SMBH mass-axion mass plane. The green region shows the observed SMBH masses at redshift $\sim 6-7$. The blue region corresponds to constraints from BH superradiance (BHSR), the gray region shows the constraints from Lyman-α forest measurements, and in the red region the wavelength of DM exceeds the smallest observed DM structures which provides a lower bound on the DM mass (see main text for more details). The orange and purple regions provide two benchmark scenarios for the relation between the axion mass and the primordial SMBH mass given in Eq. (11) with $f_a = 10^{17}$ GeV (purple) and $f_a = 10^{18}$ GeV (orange). The color intensity represents a decrease in ε' from 1 to 0.01.

From eq.(11) :

(relation between the axion mass and the SMBH mass)

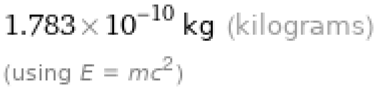
$$m_a = \varepsilon' \frac{M_P^3}{f_a M_{BH}}$$

For f_a = 1*10^17 Gev = Kg

Input interpretation

convert 1×10^{17} GeV (gigaelectronvolts) to kilograms

Result



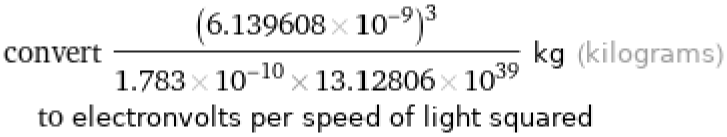
1.783×10^{-10} kg (kilograms)

(using $E = mc^2$)

We obtain:

((((((6.139608*10^-9)^3 / (1.783*10^-10*13.12806*10^39)))))) kg = eV

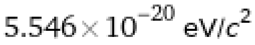
Input interpretation



$$\text{convert } \frac{(6.139608 \times 10^{-9})^3}{1.783 \times 10^{-10} \times 13.12806 \times 10^{39}} \text{ kg (kilograms)}$$

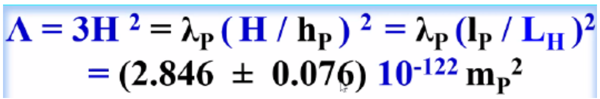
to electronvolts per speed of light squared

Result

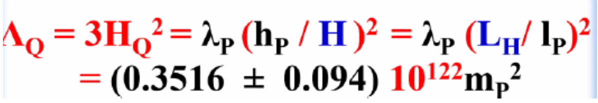


5.546×10^{-20} eV/c^2

$5.546*10^{-20}$ eV = possible Axion mass



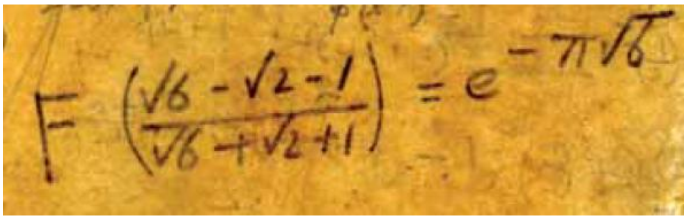
$$\Lambda = 3H^2 = \lambda_P (H / h_P)^2 = \lambda_P (l_P / L_H)^2 = (2.846 \pm 0.076)\, 10^{-122}\, m_P^2$$



$$\Lambda_Q = 3H_Q^2 = \lambda_P (h_P / H)^2 = \lambda_P (L_H / l_P)^2 = (0.3516 \pm 0.094)\, 10^{122} m_P^2$$

From:

Ramanujan Manuscript Book 1 – *Srinivasa Ramanujan*



$$F\left(\frac{\sqrt{6}-\sqrt{2}-1}{\sqrt{6}+\sqrt{2}+1}\right) = e^{-\pi\sqrt{6}}$$

For F = 1/16

1/16((sqrt6-sqr2-1)/(sqrt6+sqrt2+1))

Input

$$\frac{1}{16}\times\frac{\sqrt{6}-\sqrt{2}-1}{\sqrt{6}+\sqrt{2}+1}$$

Result

$$\frac{-1-\sqrt{2}+\sqrt{6}}{16\left(1+\sqrt{2}+\sqrt{6}\right)}$$

Decimal approximation

0.0004533091632651203706015940645516912057972002278599623166548863

…

0.000453309163….

Alternate forms

$$\frac{1}{16}\left(5-2\sqrt{6}\right)\left(7-4\sqrt{3}\right)$$

$$\frac{1}{16}+\frac{1}{-8-16\sqrt{3}+8\sqrt{6}}$$

$$\frac{1}{16}+\frac{-1-\sqrt{2}}{8\left(1+\sqrt{2}+\sqrt{6}\right)}$$

Minimal polynomial

$$65\,536\,x^4-573\,440\,x^3+75\,264\,x^2-2240\,x+1$$

Expanded forms

$$\frac{35}{16}-\frac{7\sqrt{\frac{3}{2}}}{4}+\frac{3}{\sqrt{2}}-\frac{5\sqrt{3}}{4}$$

$$-\frac{1}{16\left(1+\sqrt{2}+\sqrt{6}\right)}+\frac{\sqrt{\frac{3}{2}}}{8\left(1+\sqrt{2}+\sqrt{6}\right)}-\frac{1}{8\sqrt{2}\left(1+\sqrt{2}+\sqrt{6}\right)}$$

e^(-Pi*sqrt6)

Input

$e^{-\pi\sqrt{6}}$

Exact result

$e^{-\sqrt{6}\,\pi}$

Decimal approximation

0.0004549609435855368230139822319143761083939472675063303927388761

…

0.0004549609435….

Property

$e^{-\sqrt{6}\,\pi}$ is a transcendental number

Series representations

$$e^{-\pi\sqrt{6}}=e^{-\pi\sqrt{5}\sum_{k=0}^{\infty}5^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{6}} = \exp\left(-\pi\sqrt{5}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{5}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{6}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 5^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\operatorname{Re}(a) \text{ and } |\arg(z)| < \pi)$$

128/((1/16((sqrt6-sqr2-1)/(sqrt6+sqrt2+1))))^36

Input

$$\frac{128}{\left(\frac{1}{16} \times \frac{\sqrt{6}-\sqrt{2}-1}{\sqrt{6}+\sqrt{2}+1}\right)^{36}}$$

Exact result

$$\frac{2\,854\,495\,385\,411\,919\,762\,116\,571\,938\,898\,990\,272\,765\,493\,248\left(1+\sqrt{2}+\sqrt{6}\right)^{36}}{\left(-1-\sqrt{2}+\sqrt{6}\right)^{36}}$$

Decimal approximation

$2.999218042125173567992287390172853822670996207123453653 0641\ldots \times 10^{122}$

$2.99921804212\ldots * 10^{122}$

From which:

((128/((1/16((sqrt6-sqr2-1)/(sqrt6+sqrt2+1))))^36)) (((2 2^(4/11) 3^(6/11) log^(17/11)(3))/(e^4 log^(5/11)(2))))

where

$$\frac{2\times 2^{4/11}\times 3^{6/11}\log^{17/11}(3)}{e^4\log^{5/11}(2)}\approx 0.11723054558630\,5$$

Input

$$\frac{128}{\left(\frac{1}{16}\times\frac{\sqrt{6}-\sqrt{2}-1}{\sqrt{6}+\sqrt{2}+1}\right)^{36}}\times\frac{2\times 2^{4/11}\times 3^{6/11}\log^{17/11}(3)}{e^4\log^{5/11}(2)}$$

log(x) is the natural logarithm

Exact result

$$\left(5\,708\,990\,770\,823\,839\,524\,233\,143\,877\,797\,980\,545\,530\,986\,496\times 2^{4/11}\times 3^{6/11}\left(1+\sqrt{2}+\sqrt{6}\right)^{36}\log^{17/11}(3)\right)\Big/\left(\left(-1-\sqrt{2}+\sqrt{6}\right)^{36}e^4\log^{5/11}(2)\right)$$

Decimal approximation

$3.515999674106244680791083498070640722062869257407680001916 0\ldots\times 10^{121}$

$0.35159996741*10^{122}$

$$\Lambda_Q = 3H_Q^{\,2} = \lambda_P\,(h_P\,/\,H\,)^2 = \lambda_P\,(L_H/\,l_P)^2$$
$$= (0.3516\ \pm\ 0.094)\ 10^{122}m_P^{\,2}$$

Input interpretation

$3.5159996741\times 10^{121} = 0.35159996741\times 10^{122}$

Result

True

Expanded form

$$\begin{aligned}
&\Big(13\,300\,704\,791\,407\,472\,667\,748\,738\,114\,314\,921\,423\,207\,786\,293\,151\,170\,960\,663\ddots \\
&\qquad 663\,880\,437\,760\times 2^{4/11}\sqrt[22]{3}\,\log^{17/11}(3)\Big)\Big/ \\
&\quad\Big(\big(-1-\sqrt{2}+\sqrt{6}\big)^{36} e^4 \log^{5/11}(2)\Big)+ \\
&\Big(9\,405\,018\,454\,167\,656\,415\,788\,063\,531\,877\,787\,760\,642\,736\,514\,178\,532\,834\,803\ddots \\
&\qquad 132\,179\,415\,040\times 2^{19/22}\sqrt[22]{3}\,\log^{17/11}(3)\Big)\Big/ \\
&\quad\Big(\big(-1-\sqrt{2}+\sqrt{6}\big)^{36} e^4 \log^{5/11}(2)\Big)+ \\
&\Big(7\,679\,165\,491\,731\,217\,651\,412\,252\,111\,941\,806\,372\,273\,927\,285\,968\,760\,318\,320\ddots \\
&\qquad 876\,982\,370\,304\times 2^{4/11}\times 3^{6/11}\log^{17/11}(3)\Big)\Big/ \\
&\quad\Big(\big(-1-\sqrt{2}+\sqrt{6}\big)^{36} e^4 \log^{5/11}(2)\Big)+ \\
&\Big(5\,429\,989\,936\,246\,834\,128\,870\,645\,932\,861\,386\,501\,128\,184\,606\,730\,495\,892\,517\ddots \\
&\qquad 322\,364\,026\,880\times 2^{19/22}\times 3^{6/11}\log^{17/11}(3)\Big)\Big/ \\
&\quad\Big(\big(-1-\sqrt{2}+\sqrt{6}\big)^{36} e^4 \log^{5/11}(2)\Big)
\end{aligned}$$

Alternative representations

$$\frac{\left(2\times 2^{4/11}\times 3^{6/11}\log^{17/11}(3)\right)128}{\left(e^4\log^{5/11}(2)\right)\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^{36}}=\frac{256\times 2^{4/11}\times 3^{6/11}\log_e^{17/11}(3)}{\left(e^4\log_e^{5/11}(2)\right)\left(\frac{-1-\sqrt{2}+\sqrt{6}}{16\left(1+\sqrt{2}+\sqrt{6}\right)}\right)^{36}}$$

$$\frac{\left(2\times 2^{4/11}\times 3^{6/11}\log^{17/11}(3)\right)128}{\left(e^4\log^{5/11}(2)\right)\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^{36}}=\frac{256\times 2^{4/11}\times 3^{6/11}\left(\log(a)\log_a(3)\right)^{17/11}}{\left(e^4\left(\log(a)\log_a(2)\right)^{5/11}\right)\left(\frac{-1-\sqrt{2}+\sqrt{6}}{16\left(1+\sqrt{2}+\sqrt{6}\right)}\right)^{36}}$$

$$\frac{\left(2\times 2^{4/11}\times 3^{6/11}\log^{17/11}(3)\right)128}{\left(e^4\log^{5/11}(2)\right)\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^{36}}=\frac{256\times 2^{4/11}\times 3^{6/11}\left(2\coth^{-1}(2)\right)^{17/11}}{\left(e^4\left(2\coth^{-1}(3)\right)^{5/11}\right)\left(\frac{-1-\sqrt{2}+\sqrt{6}}{16\left(1+\sqrt{2}+\sqrt{6}\right)}\right)^{36}}$$

Or also:

((((1/16((sqrt6-sqr2-1)/(sqrt6+sqrt2+1))))^36))*(Pi^2/148)

Input

$$\left(\frac{1}{16}\times\frac{\sqrt{6}-\sqrt{2}-1}{\sqrt{6}+\sqrt{2}+1}\right)^{36}\times\frac{\pi^2}{148}$$

Result

$$\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^{36}\pi^2}{3\,300\,510\,289\,382\,532\,224\,947\,286\,304\,351\,957\,502\,885\,101\,568\left(1+\sqrt{2}+\sqrt{6}\right)^{36}}$$

Decimal approximation

$2.846033184892922680666457220879674628414026260155954529791\ldots\times 10^{-122}$

$2.8460331848\ldots*10^{-122}$

$$\Lambda = 3H^2 = \lambda_P\,(H/h_P)^2 = \lambda_P\,(l_P/L_H)^2 = (2.846 \pm 0.076)\,10^{-122}\,m_P^2$$

Property

$$\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^{36}\pi^2}{3\,300\,510\,289\,382\,532\,224\,947\,286\,304\,351\,957\,502\,885\,101\,568\left(1+\sqrt{2}+\sqrt{6}\right)^{36}}$$

is a transcendental number

((1/16((sqrt6-sqr2-1)/(sqrt6+sqrt2+1))))^6 * 2Pi

Input

$$\left(\frac{1}{16}\times\frac{\sqrt{6}-\sqrt{2}-1}{\sqrt{6}+\sqrt{2}+1}\right)^{6}\times 2\pi$$

Result

$$\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6 \pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}$$

Decimal approximation

$5.451887136876007042217740155646704560991872320643868507844 8\ldots \times 10^{-20}$

5.451887136876….$*10^{-20}$ result quite near to the value 5.546$*10^{-20}$ eV = possible Axion mass

Property

$$\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6 \pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6} \text{ is a transcendental number}$$

Alternate forms

$$\frac{\left(470\,449-192\,060\sqrt{6}\right)\left(3\,650\,401-2\,107\,560\sqrt{3}\right)\pi}{8\,388\,608}$$

$$\frac{1}{8\,388\,608}\left(1\,717\,327\,500\,049-991\,499\,494\,440\sqrt{3}-192\,060\sqrt{6\left(26\,650\,854\,921\,601-15\,386\,878\,263\,120\sqrt{3}\right)}\right)\pi$$

Expanded form

$$\frac{3465\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}-\frac{651\sqrt{\frac{3}{2}}\,\pi}{2\,097\,152\left(1+\sqrt{2}+\sqrt{6}\right)^6}+\frac{1115\pi}{2\,097\,152\sqrt{2}\left(1+\sqrt{2}+\sqrt{6}\right)^6}-\frac{495\sqrt{3}\,\pi}{2\,097\,152\left(1+\sqrt{2}+\sqrt{6}\right)^6}$$

From which:

36[-(ln((((1/16((sqrt6-sqr2-1)/(sqrt6+sqrt2+1))))^6 * 2Pi))-4+(((√(10-2√5) -2))⁄((√5-1))))]-5/2

Input

$$36\left(-\left(\log\left(\left(\frac{1}{16}\times\frac{\sqrt{6}-\sqrt{2}-1}{\sqrt{6}+\sqrt{2}+1}\right)^6\times 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\right)-\frac{5}{2}$$

log(*x*) is the natural logarithm

Exact result

$$36\left(4-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}-\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)\right)-\frac{5}{2}$$

Decimal approximation

1728.0797961315366659622763872407212437099120783186541136007663283…

1728.0797961315….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = 8^2 * 3^3) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{283}{2}-\frac{36\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}-36\log\left(\frac{\left(1+\sqrt{2}-\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)$$

$$36\left(\frac{1}{2}\left(9+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)-\log\left(\frac{1}{8\,388\,608}\left(1\,717\,327\,500\,049-991\,499\,494\,440\sqrt{3}-192\,060\sqrt{6\left(26\,650\,854\,921\,601-15\,386\,878\,263\,120\sqrt{3}\right)}\right)\pi\right)\right)-\frac{5}{2}$$

$$36\left(4-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}+\log(8\,388\,608)-6\log\left(-1-\sqrt{2}+\sqrt{6}\right)+6\log\left(1+\sqrt{2}+\sqrt{6}\right)-\log(\pi)\right)-\frac{5}{2}$$

Expanded form

$$\frac{283}{2}+\frac{72}{\sqrt{5}-1}-\frac{36\sqrt{10-2\sqrt{5}}}{\sqrt{5}-1}-36\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)$$

Alternative representations

$$36(-1)\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-\frac{5}{2}=$$

$$-\frac{5}{2}+36\left(4-\log_e\left(2\pi\left(\frac{-1-\sqrt{2}+\sqrt{6}}{16\left(1+\sqrt{2}+\sqrt{6}\right)}\right)^6\right)-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}\right)$$

$$36(-1)\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-\frac{5}{2}=$$

$$-\frac{5}{2}+36\left(4-\log(a)\log_a\left(2\pi\left(\frac{-1-\sqrt{2}+\sqrt{6}}{16\left(1+\sqrt{2}+\sqrt{6}\right)}\right)^6\right)-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}\right)$$

$$36(-1)\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-\frac{5}{2}=$$

$$-\frac{5}{2}+36\left(4+\text{Li}_1\left(1-2\pi\left(\frac{-1-\sqrt{2}+\sqrt{6}}{16\left(1+\sqrt{2}+\sqrt{6}\right)}\right)^6\right)-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}\right)$$

Series representations

$$36(-1)\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-\frac{5}{2}=$$

$$\frac{283}{2}+\frac{72}{-1+\sqrt{5}}-\frac{36\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}+36\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{\left(1+\sqrt{2}-\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)^k}{k}$$

$$36(-1)\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-\frac{5}{2}=$$

$$\frac{283}{2}+\frac{72}{-1+\sqrt{5}}-\frac{36\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}-72\,i\,\pi\left\lfloor\frac{\arg\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}-x\right)}{2\pi}\right\rfloor-$$

$$36\log(x)+36\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}-x\right)^k x^{-k}}{k}\quad\text{for } x<0$$

$$36(-1)\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-\frac{5}{2}=$$

$$\frac{283}{2}+\frac{72}{-1+\sqrt{5}}-\frac{36\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}-72\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-$$

$$36\log(z_0)+36\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{(1+\sqrt{2}-\sqrt{6})^6\pi}{8\,388\,608(1+\sqrt{2}+\sqrt{6})^6}-z_0\right)^k z_0^{-k}}{k}$$

Integral representation

$$36(-1)\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-\frac{5}{2}=$$

$$\frac{283}{2}+\frac{72}{-1+\sqrt{5}}-\frac{36\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}-36\int_1^{\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}}\frac{1}{t}\,dt$$

((36[-(ln((((1/16((sqrt6-sqr2-1)/(sqrt6+sqrt2+1))))^6 * 2Pi))-4+(((√(10-2√5) - 2))/((√5-1))))]-5/2+1))^1/15

Input

$$\sqrt[15]{36\left(-\left(\log\left(\left(\frac{1}{16}\times\frac{\sqrt{6}-\sqrt{2}-1}{\sqrt{6}+\sqrt{2}+1}\right)^6\times 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\right)-\frac{5}{2}+1}$$

log(x) is the natural logarithm

Exact result

$$\sqrt[15]{36\left(4-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}-\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)\right)-\frac{3}{2}}$$

Decimal approximation

1.6438202862876228332508710908488758313872141197706236142897303744

...

1.64382028628….≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

Alternate forms

$$\sqrt[15]{\frac{285}{2}-\frac{36\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}-36\log\left(\frac{\left(1+\sqrt{2}-\sqrt{6}\right)^6\pi}{8388608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)}$$

$$\left(36\left(\frac{1}{2}\left(9+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)-\log\left(\frac{1}{8388608}\left(1717327500049-991499494440\sqrt{3}-192060\sqrt{6\left(26650854921601-15386878263120\sqrt{3}\right)}\right)\pi\right)\right)-\frac{3}{2}\right)\text{^}(1/15)$$

$$\left(36\left(4-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}+\log(8388608)-6\log\left(-1-\sqrt{2}+\sqrt{6}\right)+6\log\left(1+\sqrt{2}+\sqrt{6}\right)-\log(\pi)\right)-\frac{3}{2}\right)\text{^}(1/15)$$

All 15th roots of 36 (4 - (sqrt(10 - 2 sqrt(5)) - 2)/(sqrt(5) - 1) - log(((-1 - sqrt(2) + sqrt(6))^6 π)/(8388608 (1 + sqrt(2) + sqrt(6))^6))) - 3/2

$$e^{0}\sqrt[15]{36\left(4-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}-\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^{6}\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^{6}}\right)\right)-\frac{3}{2}}\approx 1.6438$$

(real, principal root)

$$e^{(2i\pi)/15}\sqrt[15]{36\left(4-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}-\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^{6}\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^{6}}\right)\right)-\frac{3}{2}}$$

$\approx 1.5017+0.6686\,i$

$$e^{(4i\pi)/15}\sqrt[15]{36\left(4-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}-\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^{6}\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^{6}}\right)\right)-\frac{3}{2}}$$

$\approx 1.0999+1.2216\,i$

$$e^{(2i\pi)/5}\sqrt[15]{36\left(4-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}-\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^{6}\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^{6}}\right)\right)-\frac{3}{2}}$$

$\approx 0.5080+1.5634\,i$

$$e^{(8i\pi)/15}\sqrt[15]{36\left(4-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}-\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^{6}\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^{6}}\right)\right)-\frac{3}{2}}$$

$\approx -0.17183+1.6348\,i$

Alternative representations

$$\sqrt[15]{36(-1)\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-\frac{5}{2}+1} =$$

$$\sqrt[15]{-\frac{3}{2}+36\left(4-\log_e\left(2\pi\left(\frac{-1-\sqrt{2}+\sqrt{6}}{16\left(1+\sqrt{2}+\sqrt{6}\right)}\right)^6\right)-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}\right)}$$

$$\sqrt[15]{36(-1)\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-\frac{5}{2}+1} =$$

$$\sqrt[15]{-\frac{3}{2}+36\left(4-\log(a)\log_a\left(2\pi\left(\frac{-1-\sqrt{2}+\sqrt{6}}{16\left(1+\sqrt{2}+\sqrt{6}\right)}\right)^6\right)-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}\right)}$$

$$\sqrt[15]{36(-1)\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-\frac{5}{2}+1} =$$

$$\sqrt[15]{-\frac{3}{2}+36\left(4+\mathrm{Li}_1\left(1-2\pi\left(\frac{-1-\sqrt{2}+\sqrt{6}}{16\left(1+\sqrt{2}+\sqrt{6}\right)}\right)^6\right)-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}\right)}$$

Series representations

$$\sqrt[15]{36(-1)\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-\frac{5}{2}+1} =$$

$$\sqrt[15]{\frac{285}{2}-\frac{36\left(-2+\sqrt{10-2\sqrt{5}}\right)}{-1+\sqrt{5}}+36\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{\left(1+\sqrt{2}-\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)^k}{k}}$$

$$\sqrt[15]{36(-1)\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-\frac{5}{2}+1} =$$

$$\sqrt[15]{-\frac{3}{2}+36\left(4-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}+\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{\left(1+\sqrt{2}-\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)^k}{k}\right)}$$

$$\sqrt[15]{36(-1)\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-\frac{5}{2}+1} =$$

$$\left(-\frac{3}{2}+36\left(4-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}-2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-\right.\right.$$
$$\left.\left.\log(z_0)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{\left(1+\sqrt{2}-\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}-z_0\right)^k z_0^{-k}}{k}\right)\right)\text{^}(1/15)$$

Integral representation

$$\sqrt[15]{36(-1)\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-\frac{5}{2}+1} =$$

$$\sqrt[15]{-\frac{3}{2}+36\left(4-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}-\int_1^{\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}}\frac{1}{t}\,dt\right)}$$

And again:

(1/27(36[-(ln((((1/16((sqrt6-sqr2-1)/(sqrt6+sqrt2+1))))^6 * 2Pi))-4+(((√(10-2√5) -2))/((√5-1))))]-5/2))^2-(((√(10-2√5) -2))/((√5-1)))

Input

$$\left(\frac{1}{27}\left(36\left(-\left(\log\left(\left(\frac{1}{16}\times\frac{\sqrt{6}-\sqrt{2}-1}{\sqrt{6}+\sqrt{2}+1}\right)^6\times 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\right)-\frac{5}{2}\right)\right)^2-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

log(*x*) is the natural logarithm

Exact result

$$\frac{1}{729}\left(36\left(4-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}-\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)\right)-\frac{5}{2}\right)^2-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

Decimal approximation

4096.0942224623507067472136017335830315043577316416424353975472872
…

4096.09422246235….≈ 4096 = 64^2

Alternate forms

$$\frac{1}{2}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)+\frac{1}{729}\left(36\left(\frac{1}{2}\left(9+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)-\log\left(\frac{1}{8\,388\,608}\left(1\,717\,327\,500\,049-991\,499\,494\,440\sqrt{3}-192\,060\sqrt{6\left(26\,650\,854\,921\,601-15\,386\,878\,263\,120\sqrt{3}\right)}\right)\pi\right)\right)-\frac{5}{2}\right)^2$$

$$\frac{77\,629}{486\left(\sqrt{5}-1\right)^2}-\frac{41\,605\sqrt{5}}{1458\left(\sqrt{5}-1\right)^2}+\frac{637\sqrt{2\left(5-\sqrt{5}\right)}}{81\left(\sqrt{5}-1\right)^2}-\frac{1213\sqrt{10\left(5-\sqrt{5}\right)}}{81\left(\sqrt{5}-1\right)^2}-\frac{32\left(\sqrt{5}-3\right)\log^2\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)}{9\left(\sqrt{5}-1\right)^2}+\left(-\frac{2072}{27\left(\sqrt{5}-1\right)^2}+\frac{1688\sqrt{5}}{81\left(\sqrt{5}-1\right)^2}-\frac{32\sqrt{2\left(5-\sqrt{5}\right)}}{9\left(\sqrt{5}-1\right)^2}+\frac{32\sqrt{10\left(5-\sqrt{5}\right)}}{9\left(\sqrt{5}-1\right)^2}\right)\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)$$

$$\frac{1}{1458\left(\sqrt{5}-1\right)^2}\left(232887-41605\sqrt{5}+11466\sqrt{2\left(5-\sqrt{5}\right)}-\right.$$
$$21834\sqrt{10\left(5-\sqrt{5}\right)}+15552\log^2\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8388608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)-$$
$$5184\sqrt{5}\log^2\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8388608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)-$$
$$111888\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8388608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)+$$
$$30384\sqrt{5}\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8388608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)-$$
$$5184\sqrt{2\left(5-\sqrt{5}\right)}\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8388608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)+$$
$$\left.5184\sqrt{10\left(5-\sqrt{5}\right)}\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8388608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)\right)$$

$$\frac{1}{729}\left(\frac{5}{2}-36\left(4+\frac{2-\sqrt{10-2\sqrt{5}}}{\sqrt{5}-1}+23\log(2)-6\log\left(-1-\sqrt{2}+\sqrt{6}\right)+\right.\right.$$
$$\left.\left.6\log\left(1+\sqrt{2}+\sqrt{6}\right)-\log(\pi)\right)\right)^2-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

Expanded form

$$\frac{80089}{2916}+\frac{224}{9\left(\sqrt{5}-1\right)^2}-\frac{32\sqrt{5}}{9\left(\sqrt{5}-1\right)^2}-$$
$$\frac{64\sqrt{10-2\sqrt{5}}}{9\left(\sqrt{5}-1\right)^2}+\frac{2426}{81\left(\sqrt{5}-1\right)}-\frac{1213\sqrt{10-2\sqrt{5}}}{81\left(\sqrt{5}-1\right)}+$$
$$\frac{16}{9}\log^2\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8388608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)-\frac{1132}{81}\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8388608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)-$$
$$\frac{64\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8388608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)}{9\left(\sqrt{5}-1\right)}+\frac{32\sqrt{10-2\sqrt{5}}\log\left(\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8388608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)}{9\left(\sqrt{5}-1\right)}$$

Alternative representations

$$\left(\frac{1}{27}\left(36\left(-\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^{6} 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\right)-\frac{5}{2}\right)\right)^{2}-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}=$$

$$\left(\frac{1}{27}\left(-\frac{5}{2}+36\left(4-\log_{e}\left(2\pi\left(\frac{-1-\sqrt{2}+\sqrt{6}}{16\left(1+\sqrt{2}+\sqrt{6}\right)}\right)^{6}\right)-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}\right)\right)\right)^{2}-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}$$

$$\left(\frac{1}{27}\left(36\left(-\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^{6} 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\right)-\frac{5}{2}\right)\right)^{2}-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}=$$

$$\left(\frac{1}{27}\left(-\frac{5}{2}+36\left(4-\log(a)\log_{a}\left(2\pi\left(\frac{-1-\sqrt{2}+\sqrt{6}}{16\left(1+\sqrt{2}+\sqrt{6}\right)}\right)^{6}\right)-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}\right)\right)\right)^{2}-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}$$

$$\left(\frac{1}{27}\left(36\left(-\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^{6} 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\right)-\frac{5}{2}\right)\right)^{2}-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}=$$

$$\left(\frac{1}{27}\left(-\frac{5}{2}+36\left(4+\mathrm{Li}_{1}\left(1-2\pi\left(\frac{-1-\sqrt{2}+\sqrt{6}}{16\left(1+\sqrt{2}+\sqrt{6}\right)}\right)^{6}\right)-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}\right)\right)\right)^{2}-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}$$

Series representations

$$\left(\frac{1}{27}\left(36\left(-\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\right)-\frac{5}{2}\right)\right)^2-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}=-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}+\frac{1}{729}\left(-\frac{5}{2}+36\left(4-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}+\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{\left(1+\sqrt{2}-\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)^k}{k}\right)\right)^2$$

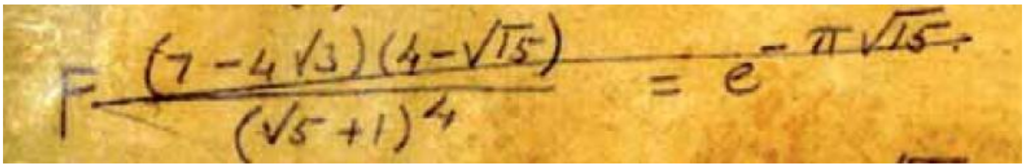
$$\left(\frac{1}{27}\left(36\left(-\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\right)-\frac{5}{2}\right)\right)^2-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}=-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}+\frac{1}{729}\left(-\frac{5}{2}+36\left(4-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}+\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)^k}{k}\right)\right)^2$$

$$\left(\frac{1}{27}\left(36\left(-\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\right)-\frac{5}{2}\right)\right)^2-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}=-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}+\frac{1}{729}\left(-\frac{283}{2}+\frac{36\left(-2+\sqrt{10-2\sqrt{5}}\right)}{-1+\sqrt{5}}-36\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{\left(1+\sqrt{2}-\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}\right)^k}{k}\right)^2$$

Integral representation

$$\left(\frac{1}{27}\left(36\left(-\left(\log\left(\left(\frac{\sqrt{6}-\sqrt{2}-1}{16\left(\sqrt{6}+\sqrt{2}+1\right)}\right)^6 2\pi\right)-4+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\right)-\frac{5}{2}\right)\right)^2-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}=-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}+\frac{1}{729}\left(-\frac{5}{2}+36\left(4-\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}-\int_1^{\frac{\left(-1-\sqrt{2}+\sqrt{6}\right)^6\pi}{8\,388\,608\left(1+\sqrt{2}+\sqrt{6}\right)^6}}\frac{1}{t}\,dt\right)\right)^2$$

From:



$$F\frac{(7-4\sqrt{3})(4-\sqrt{15})}{(\sqrt{5}+1)^4}=e^{-\pi\sqrt{15}}.$$

We have:

((7-4sqrt3)(4-sqrt15))/(sqrt5+1)^4

Input

$$\frac{\left(7-4\sqrt{3}\right)\left(4-\sqrt{15}\right)}{\left(\sqrt{5}+1\right)^4}$$

Decimal approximation

0.0000831562753263143790190690523904318927325474386170242819305430
…

0.0000831562753…..

Alternate forms

$$\frac{1}{32}\left(7-4\sqrt{3}\right)\left(4-\sqrt{15}\right)\left(7-3\sqrt{5}\right)$$

$$-\frac{7\sqrt{3}}{32}-\frac{\sqrt{15}}{32}+\frac{1}{2}$$

$$\frac{\left(4\sqrt{3}-7\right)\left(\sqrt{15}-4\right)}{\left(1+\sqrt{5}\right)^4}$$

Minimal polynomial

$$65\,536\,x^4-131\,072\,x^3+77\,568\,x^2-12\,032\,x+1$$

Expanded forms

$$\frac{1}{2}-\frac{7\sqrt{3}}{32}-\frac{\sqrt{15}}{32}$$

$$\frac{28}{\left(1+\sqrt{5}\right)^4}-\frac{16\sqrt{3}}{\left(1+\sqrt{5}\right)^4}+\frac{12\sqrt{5}}{\left(1+\sqrt{5}\right)^4}-\frac{7\sqrt{15}}{\left(1+\sqrt{5}\right)^4}$$

And:

e^(-Pi*sqrt15)

Input

$$e^{-\pi\sqrt{15}}$$

Exact result

$$e^{-\sqrt{15}\,\pi}$$

Decimal approximation

$5.1974833123792749017671397011547340751769584788985809108151\ldots \times 10^{-6}$

$5.197483312\ldots * 10^{-6}$

Property

$e^{-\sqrt{15}\,\pi}$ is a transcendental number

Series representations

$$e^{-\pi\sqrt{15}} = e^{-\pi\sqrt{14}\sum_{k=0}^{\infty} 14^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{15}} = \exp\left(-\pi\sqrt{14}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{14}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$e^{-\pi\sqrt{15}} = \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 14^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Integral representation

$$(1+z)^{a} = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^{s}}\,ds}{(2\,\pi\,i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\operatorname{Re}(a) \text{ and } |\arg(z)|<\pi)$$

We note that:

((7-4sqrt3)(4-sqrt15))/(sqrt5+1)^4*x = e^(-Pi*sqrt15)

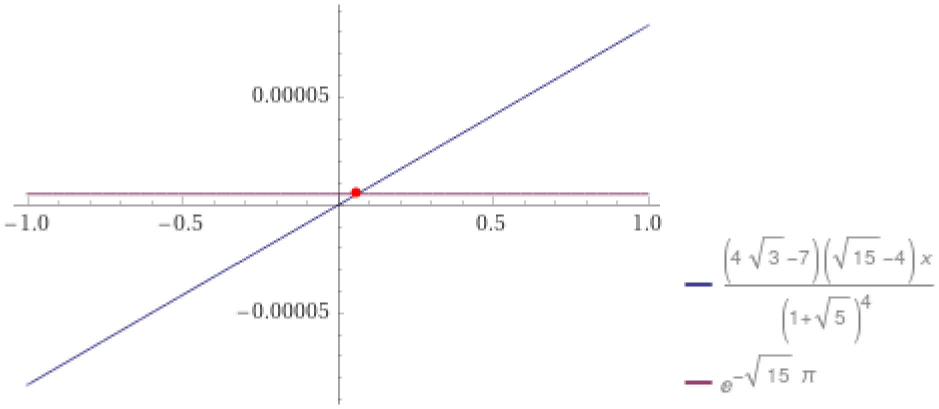
Input

$$\frac{\left(7-4\sqrt{3}\right)\left(4-\sqrt{15}\right)}{\left(\sqrt{5}+1\right)^4}\,x = e^{-\pi\sqrt{15}}$$

Exact result

$$\frac{\left(7-4\sqrt{3}\right)\left(4-\sqrt{15}\right)x}{\left(1+\sqrt{5}\right)^4} = e^{-\sqrt{15}\,\pi}$$

Plot



Alternate forms

$$\frac{\left(4\sqrt{3}-7\right)\left(\sqrt{15}-4\right)x}{\left(1+\sqrt{5}\right)^4} = e^{-\sqrt{15}\,\pi}$$

$$\frac{\left(4\sqrt{3}-7\right)\left(\sqrt{15}-4\right)x}{8\left(7+3\sqrt{5}\right)} = e^{-\sqrt{15}\,\pi}$$

$$-\frac{x}{64\left(-47-21\sqrt{5}\right)\left(1+\sqrt{1+\frac{1}{32\left(-47-21\sqrt{5}\right)}}\right)}=e^{-\sqrt{15}\,\pi}$$

Expanded forms

$$-\frac{7\sqrt{15}\,x}{\left(1+\sqrt{5}\right)^4}+\frac{12\sqrt{5}\,x}{\left(1+\sqrt{5}\right)^4}-\frac{16\sqrt{3}\,x}{\left(1+\sqrt{5}\right)^4}+\frac{28\,x}{\left(1+\sqrt{5}\right)^4}=e^{-\sqrt{15}\,\pi}$$

$$-\frac{\sqrt{15}\,x}{32}-\frac{7\sqrt{3}\,x}{32}+\frac{x}{2}=e^{-\sqrt{15}\,\pi}$$

Solution

$x \approx 0.062503$

0.062503 ≈ 1/16

Thence:

1/16*((7-4sqrt3)(4-sqrt15))/(sqrt5+1)^4

Input

$$\frac{1}{16}\times\frac{\left(7-4\sqrt{3}\right)\left(4-\sqrt{15}\right)}{\left(\sqrt{5}+1\right)^4}$$

Result

$$\frac{\left(7-4\sqrt{3}\right)\left(4-\sqrt{15}\right)}{16\left(1+\sqrt{5}\right)^4}$$

Decimal approximation

$5.1972672078946486886918157744019932957842149135640176206589\ldots \times 10^{-6}$

5.1972672078…*10^{-6}

Alternate forms

$$\frac{1}{512}\left(7-4\sqrt{3}\right)\left(4-\sqrt{15}\right)\left(7-3\sqrt{5}\right)$$

$$-\frac{7\sqrt{3}}{512}-\frac{\sqrt{15}}{512}+\frac{1}{32}$$

$$\frac{\left(4\sqrt{3}-7\right)\left(\sqrt{15}-4\right)}{16\left(1+\sqrt{5}\right)^4}$$

Minimal polynomial

$$4294967296\,x^4-536870912\,x^3+19857408\,x^2-192512\,x+1$$

Expanded forms

$$\frac{1}{32}-\frac{7\sqrt{3}}{512}-\frac{\sqrt{15}}{512}$$

$$\frac{7}{4\left(1+\sqrt{5}\right)^4}-\frac{\sqrt{3}}{\left(1+\sqrt{5}\right)^4}+\frac{3\sqrt{5}}{4\left(1+\sqrt{5}\right)^4}-\frac{7\sqrt{15}}{16\left(1+\sqrt{5}\right)^4}$$

From which:

(((1/16*((7-4sqrt3)(4-sqrt15))/(sqrt5+1)^4)))^23 * e/log(16)

Input

$$\left(\frac{1}{16} \times \frac{(7-4\sqrt{3})(4-\sqrt{15})}{(\sqrt{5}+1)^4}\right)^{23} \times \frac{e}{\log(16)}$$

log(x) is the natural logarithm

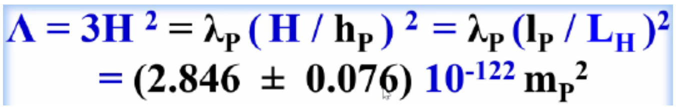
Exact result

$$\frac{(7-4\sqrt{3})^{23}(4-\sqrt{15})^{23}\, e}{4951760157141521099596496896\,(1+\sqrt{5})^{92}\log(16)}$$

Decimal approximation

$2.846001233498706308020164492840338767382071632560690149022\ldots \times 10^{-122}$

$2.84600123349\ldots{*}10^{-122}$



$$\Lambda = 3H^2 = \lambda_P (H / h_P)^2 = \lambda_P (l_P / L_H)^2 = (2.846 \pm 0.076)\, 10^{-122}\, m_P^2$$

Alternate forms

$$\frac{(4\sqrt{3}-7)^{23}(\sqrt{15}-4)^{23}\, e}{4951760157141521099596496896\,(1+\sqrt{5})^{92}\log(16)}$$

$$\left(\left(2\,475\,358\,407\,258\,740\,568\,048\,819\,256 - 1\,429\,148\,842\,771\,617\,655\,131\,742\,823\sqrt{3} - \sqrt{\left(15\left(811\,438\,864\,107\,456\,432\,540\,037\,334\,094\,673\,010\,837\,125\,725\,872\,884\,993 - 468\,484\,446\,623\,364\,131\,191\,582\,620\,295\,042\,125\,957\,800\,711\,469\,088\,784\sqrt{3}\right)\right)}\right)e\right) \Big/ \left(49\,039\,857\,307\,708\,443\,467\,467\,104\,868\,809\,893\,875\,799\,651\,909\,875\,269\,632\log(16)\right)$$

$$\frac{\left(4\sqrt{3}-7\right)^{23}\left(\sqrt{15}-4\right)^{23}e}{19\,807\,040\,628\,566\,084\,398\,385\,987\,584\left(1+\sqrt{5}\right)^{92}\log(2)}$$

$$\frac{\left(7-4\sqrt{3}\right)^{23}\left(4-\sqrt{15}\right)^{23}e}{19\,807\,040\,628\,566\,084\,398\,385\,987\,584\left(1+\sqrt{5}\right)^{92}\log(2)}$$

Expanded forms

$$\frac{5\,208\,010\,578\,819\,886\,397\,394\,028\,748\,702\,579\,844\,626\,346\,847\,e}{1\,237\,940\,039\,285\,380\,274\,899\,124\,224\left(1+\sqrt{5}\right)^{92}\log(16)} - \frac{751\,711\,577\,406\,020\,025\,416\,334\,702\,150\,914\,230\,725\,419\,759\sqrt{3}\,e}{309\,485\,009\,821\,345\,068\,724\,781\,056\left(1+\sqrt{5}\right)^{92}\log(16)} + \frac{2\,329\,093\,136\,355\,858\,490\,014\,967\,958\,525\,269\,261\,432\,627\,131\sqrt{5}\,e}{1\,237\,940\,039\,285\,380\,274\,899\,124\,224\left(1+\sqrt{5}\right)^{92}\log(16)} - \frac{5\,378\,810\,196\,971\,058\,582\,122\,265\,347\,907\,937\,530\,766\,702\,841\sqrt{15}\,e}{4\,951\,760\,157\,141\,521\,099\,596\,496\,896\left(1+\sqrt{5}\right)^{92}\log(16)}$$

$$\frac{309\,419\,800\,907\,342\,571\,006\,102\,407\,e}{6\,129\,982\,163\,463\,555\,433\,433\,388\,108\,601\,236\,734\,474\,956\,488\,734\,408\,704\log(16)} - \left(1\,429\,148\,842\,771\,617\,655\,131\,742\,823\sqrt{3}\;e\right)/ (49\,039\,857\,307\,708\,443\,467\,467\,104\,868\,809\,893\,875\,799\,651\,909\,875\,269\,632\log(16)) + \left(137\,906\,121\,610\,022\,264\,554\,466\,397\sqrt{5}\;e\right)/ (6\,129\,982\,163\,463\,555\,433\,433\,388\,108\,601\,236\,734\,474\,956\,488\,734\,408\,704\log(16)) - \left(636\,961\,091\,475\,557\,688\,577\,270\,751\sqrt{15}\;e\right)/ (49\,039\,857\,307\,708\,443\,467\,467\,104\,868\,809\,893\,875\,799\,651\,909\,875\,269\,632\log(16))$$

Alternative representations

$$\frac{\left(\frac{(7-4\sqrt{3})(4-\sqrt{15})}{16(\sqrt{5}+1)^4}\right)^{23} e}{\log(16)} = \frac{e\left(\frac{(7-4\sqrt{3})(4-\sqrt{15})}{16(1+\sqrt{5})^4}\right)^{23}}{\log_e(16)}$$

$$\frac{\left(\frac{(7-4\sqrt{3})(4-\sqrt{15})}{16(\sqrt{5}+1)^4}\right)^{23} e}{\log(16)} = \frac{e\left(\frac{(7-4\sqrt{3})(4-\sqrt{15})}{16(1+\sqrt{5})^4}\right)^{23}}{\log(a)\log_a(16)}$$

$$\frac{\left(\frac{(7-4\sqrt{3})(4-\sqrt{15})}{16(\sqrt{5}+1)^4}\right)^{23} e}{\log(16)} = -\frac{e\left(\frac{(7-4\sqrt{3})(4-\sqrt{15})}{16(1+\sqrt{5})^4}\right)^{23}}{\mathrm{Li}_1(-15)}$$

Series representations

$$\frac{\left(\frac{(7-4\sqrt{3})(4-\sqrt{15})}{16(\sqrt{5}+1)^4}\right)^{23} e}{\log(16)} = \frac{\left(-7+4\sqrt{3}\right)^{23}\left(-4+\sqrt{15}\right)^{23} e}{4\,951\,760\,157\,141\,521\,099\,596\,496\,896\left(1+\sqrt{5}\right)^{92}\left(\log(15)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{15}\right)^k}{k}\right)}$$

$$\frac{\left(\frac{(7-4\sqrt{3})(4-\sqrt{15})}{16(\sqrt{5}+1)^4}\right)^{23} e}{\log(16)} = \frac{(7-4\sqrt{3})^{23}(4-\sqrt{15})^{23} e}{4\,951\,760\,157\,141\,521\,099\,596\,496\,896(1+\sqrt{5})^{92}\left(\log(15)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{15}\right)^k}{k}\right)}$$

$$\frac{\left(\frac{(7-4\sqrt{3})(4-\sqrt{15})}{16(\sqrt{5}+1)^4}\right)^{23} e}{\log(16)} = \left((-7+4\sqrt{3})^{23}(-4+\sqrt{15})^{23} e\right)\Big/ \left(4\,951\,760\,157\,141\,521\,099\,596\,496\,896(1+\sqrt{5})^{92} \left(2 i \pi \left\lfloor \frac{\arg(16-x)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{(-1)^k (16-x)^k x^{-k}}{k}\right)\right) \text{ for } x < 0$$

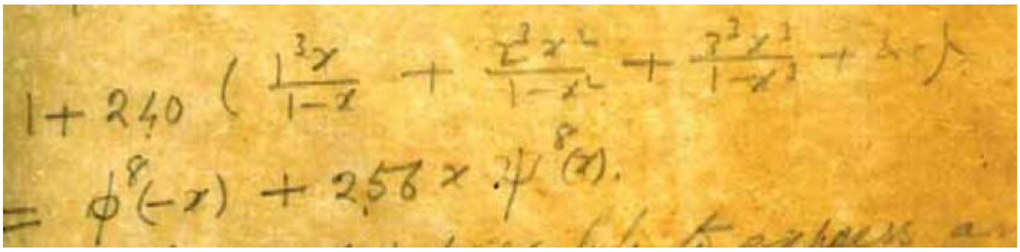
Integral representations

$$\frac{\left(\frac{(7-4\sqrt{3})(4-\sqrt{15})}{16(\sqrt{5}+1)^4}\right)^{23} e}{\log(16)} = \frac{(-7+4\sqrt{3})^{23}(-4+\sqrt{15})^{23} e}{4\,951\,760\,157\,141\,521\,099\,596\,496\,896(1+\sqrt{5})^{92}\int_1^{16}\frac{1}{t}\,dt}$$

$$\frac{\left(\frac{(7-4\sqrt{3})(4-\sqrt{15})}{16(\sqrt{5}+1)^4}\right)^{23} e}{\log(16)} = \frac{i(-7+4\sqrt{3})^{23}(-4+\sqrt{15})^{23} e\,\pi}{2\,475\,880\,078\,570\,760\,549\,798\,248\,448(1+\sqrt{5})^{92}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{15^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}$$

for $-1 < \gamma < 0$

From:



$$1+240\left(\frac{1^3x}{1-x}+\frac{2^3x^2}{1-x^2}+\frac{3^3x^3}{1-x^3}+\&c\right)$$

$$=\phi^8(-x)+256x\,\psi^8(x).$$

1+240(x/(1-x)+8x^2/(1-x^2)+27x^3/(1-x^3)) = φ^8(-x) + 256x*ψ^8(x)

Input

$$1+240\left(\frac{x}{1-x}+8\times\frac{x^2}{1-x^2}+27\times\frac{x^3}{1-x^3}\right)=\phi(-x)^8+256\,x\,\psi(x)^8$$

$\phi(n)$ is the Euler totient function

$\psi(x)$ is the digamma function

Alternate forms

$$1-\frac{240\,x\,(x\,(x\,(36\,x+37)+10)+1)}{x^4+x^3-x-1}=\phi(-x)^8+256\,x\,\psi^{(0)}(x)^8$$

$$1-\frac{240\,x\left(36\,x^3+37\,x^2+10\,x+1\right)}{x^4+x^3-x-1}=\phi(-x)^8+256\,x\,\psi^{(0)}(x)^8$$

$$-\frac{8639\,x^4+8879\,x^3+2400\,x^2+241\,x+1}{(x-1)\,(x+1)\left(x^2+x+1\right)}=\phi(-x)^8+256\,x\,\psi^{(0)}(x)^8$$

1 - (240 x (x (x (36 x + 37) + 10) + 1))/(x^4 + x^3 - x - 1) = ϕ(-x)^8 + 256 x polygamma(0, x)^8

Input

$$1-\frac{240\,x\,(x\,(x\,(36\,x+37)+10)+1)}{x^4+x^3-x-1}=\phi(-x)^8+256\,x\,\psi^{(0)}(x)^8$$

$\phi(n)$ is the Euler totient function

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Alternate form

$$-\frac{8639\,x^4+8879\,x^3+2400\,x^2+241\,x+1}{(x-1)\,(x+1)\left(x^2+x+1\right)}=\phi(-x)^8+256\,x\,\psi^{(0)}(x)^8$$

Expanded form

$$-\frac{8640\,x^4}{x^4+x^3-x-1}-\frac{8880\,x^3}{x^4+x^3-x-1}-\frac{240\,x}{x^4+x^3-x-1}-\frac{2400\,x^2}{x^4+x^3-x-1}+1=\phi(-x)^8+256\,x\,\psi^{(0)}(x)^8$$

-(8640 x^4)/(x^4 + x^3 - x - 1) - (8880 x^3)/(x^4 + x^3 - x - 1) - (240 x)/(x^4 + x^3 - x - 1) - (2400 x^2)/(x^4 + x^3 - x - 1) + 1 = ϕ(-x)^8 + 256 x polygamma(0, x)^8

Input

$$-\frac{8640\,x^4}{x^4+x^3-x-1}-\frac{8880\,x^3}{x^4+x^3-x-1}-\frac{240\,x}{x^4+x^3-x-1}-\frac{2400\,x^2}{x^4+x^3-x-1}+1=\phi(-x)^8+256\,x\,\psi^{(0)}(x)^8$$

$\phi(n)$ is the Euler totient function

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Alternate forms

$$\frac{2160\,(x+2)}{x^2+x+1}-\frac{3360}{x-1}+\frac{960}{x+1}-8639=\phi(-x)^8+256\,x\,\psi^{(0)}(x)^8$$

$$\frac{8639\,x^4+8879\,x^3+2400\,x^2+241\,x+1}{-x^4-x^3+x+1}=\phi(-x)^8+256\,x\,\psi^{(0)}(x)^8$$

$$-\frac{8639\,x^4+8879\,x^3+2400\,x^2+241\,x+1}{(x-1)\,(x+1)\left(x^2+x+1\right)}=\phi(-x)^8+256\,x\,\psi^{(0)}(x)^8$$

(-(8639 x^4 + 8879 x^3 + 2400 x^2 + 241 x + 1)/((x - 1) (x + 1) (x^2 + x + 1)))

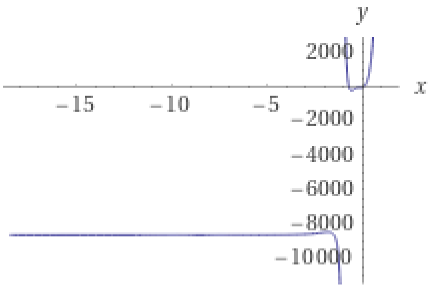
Input

$$-\frac{8639\,x^4+8879\,x^3+2400\,x^2+241\,x+1}{(x-1)\,(x+1)\left(x^2+x+1\right)}$$

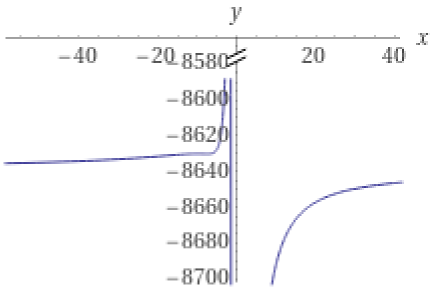
Result

$$\frac{-8639\,x^4-8879\,x^3-2400\,x^2-241\,x-1}{(x-1)\,(x+1)\left(x^2+x+1\right)}$$

Plots



(x from −18.5 to 1.6)



(x from −58.6 to 41.7)

Alternate forms

$$-\frac{8639\,x^4}{x^4+x^3-x-1}-\frac{8879\,x^3}{x^4+x^3-x-1}-\frac{241\,x}{x^4+x^3-x-1}-\frac{1}{x^4+x^3-x-1}-\frac{2400\,x^2}{x^4+x^3-x-1}$$

$$\frac{2160\,(x+2)}{x^2+x+1}-\frac{3360}{x-1}+\frac{960}{x+1}-8639$$

$$\frac{x\,(x\,((-8639\,x-8879)\,x-2400)-241)-1}{x\,\left(x^2\,(x+1)-1\right)-1}$$

Expanded forms

$$-\frac{2400\,x^2}{(x-1)\,(x+1)\left(x^2+x+1\right)}-\frac{241\,x}{(x-1)\,(x+1)\left(x^2+x+1\right)}-\frac{1}{(x-1)\,(x+1)\left(x^2+x+1\right)}-\frac{8639\,x^4}{(x-1)\,(x+1)\left(x^2+x+1\right)}-\frac{8879\,x^3}{(x-1)\,(x+1)\left(x^2+x+1\right)}$$

$$\frac{-8639\,x^4-8879\,x^3-2400\,x^2-241\,x-1}{x^4+x^3-x-1}$$

Real roots

$x \approx -0.67861$

$x \approx -0.0043334$

Properties as a real function

Domain

$\{x \in \mathbb{R} : x \neq -1$ and $x \neq 1\}$

Range

$\{y \in \mathbb{R} : y \leq -8461.95$ or $y + 4319 \geq 4142.95\}$

$\mathbb{R}$ is the set of real numbers

Series expansion at x=0

$$1 + 240\,x + 2160\,x^2 + 6720\,x^3 + 2160\,x^4 + O\left(x^5\right)$$

(Taylor series)

Series expansion at x=∞

$$-8639 - \frac{240}{x} - \frac{2160}{x^2} - \frac{6720}{x^3} + O\left(\left(\frac{1}{x}\right)^4\right)$$

(Laurent series)

Derivative

$$\frac{d}{dx}\left(-\frac{8639\,x^4 + 8879\,x^3 + 2400\,x^2 + 241\,x + 1}{(x-1)\,(x+1)\left(x^2 + x + 1\right)}\right) = \frac{240\left(x^6 + 20\,x^5 + 121\,x^4 + 220\,x^3 + 121\,x^2 + 20\,x + 1\right)}{\left(x^4 + x^3 - x - 1\right)^2}$$

Indefinite integral

$$\int \frac{-1-241\,x-2400\,x^2-8879\,x^3-8639\,x^4}{(-1+x)\,(1+x)\left(1+x+x^2\right)}\,dx = 1080\log\left(x^2+x+1\right)-8639\,x-3360\log(1-x)+960\log(x+1)+2160\sqrt{3}\,\tan^{-1}\left(\frac{2\,x+1}{\sqrt{3}}\right)+\text{constant}$$

(assuming a complex-valued logarithm)

From:

$$\frac{-8639\,x^4-8879\,x^3-2400\,x^2-241\,x-1}{(x-1)\,(x+1)\left(x^2+x+1\right)}$$

For x = -0.0043334

((-(8639 (-0.0043334)^4 + 8879 (-0.0043334)^3 + 2400 (-0.0043334)^2 + 241 (-0.0043334) + 1)/(((-0.0043334) - 1) ((-0.0043334) + 1) ((-0.0043334)^2 + (-0.0043334) + 1))))^3 * (-(tan((17 π)/542)))

where

$$\tan\left(\frac{17\,\pi}{542}\right) \approx 0.09885719915$$

Input

$$\left(-\left(\left(8639\,(-0.0043334)^4+8879\,(-0.0043334)^3+2400\,(-0.0043334)^2+241\times(-0.0043334)+1\right)\Big/\left((-0.0043334-1)\,(-0.0043334+1)\left((-0.0043334)^2-0.0043334+1\right)\right)\right)\right)^3\left(-\tan\left(\frac{17\,\pi}{542}\right)\right)$$

Result

$5.54600\ldots\times10^{-20}$

$5.546\ldots*10^{-20}$ result practically equal to $5.546*10^{-20}$ eV = possible Axion mass

Alternative representations

$$\left(-\left(\left(8639\,(-0.0043334)^4 + 8879\,(-0.0043334)^3 + 2400\,(-0.0043334)^2 + 241\,(-0.0043334) + 1\right)\Big/\left((-0.0043334 - 1)\,(-0.0043334 + 1)\left((-0.0043334)^2 - 0.0043334 + 1\right)\right)\right)\right)^3 (-1)\tan\left(\frac{17\pi}{542}\right) = -\frac{\left(-\frac{0.0443494 - 2400\,(-0.0043334)^2 - 8879\,(-0.0043334)^3 - 8639\,(-0.0043334)^4}{0.999981\left(0.995667 + (-0.0043334)^2\right)}\right)^3}{\cot\left(\frac{17\pi}{542}\right)}$$

$$\left(-\left(\left(8639\,(-0.0043334)^4 + 8879\,(-0.0043334)^3 + 2400\,(-0.0043334)^2 + 241\,(-0.0043334) + 1\right)\Big/\left((-0.0043334 - 1)\,(-0.0043334 + 1)\left((-0.0043334)^2 - 0.0043334 + 1\right)\right)\right)\right)^3 (-1)\tan\left(\frac{17\pi}{542}\right) = \cot\left(\frac{\pi}{2} + \frac{17\pi}{542}\right)\left(-\left(\left(0.0443494 - 2400\,(-0.0043334)^2 - 8879\,(-0.0043334)^3 - 8639\,(-0.0043334)^4\right)\Big/\left(0.999981\left(0.995667 + (-0.0043334)^2\right)\right)\right)\right)^3$$

$$\left(-\left(\left(8639\,(-0.0043334)^4 + 8879\,(-0.0043334)^3 + 2400\,(-0.0043334)^2 + 241\,(-0.0043334) + 1\right)\Big/\left((-0.0043334 - 1)\,(-0.0043334 + 1)\left((-0.0043334)^2 - 0.0043334 + 1\right)\right)\right)\right)^3 (-1)\tan\left(\frac{17\pi}{542}\right) = -\cot\left(\frac{\pi}{2} - \frac{17\pi}{542}\right)\left(-\left(\left(0.0443494 - 2400\,(-0.0043334)^2 - 8879\,(-0.0043334)^3 - 8639\,(-0.0043334)^4\right)\Big/\left(0.999981\left(0.995667 + (-0.0043334)^2\right)\right)\right)\right)^3$$

Series representations

$$\left(-\left(\left(8639\,(-0.0043334)^4 + 8879\,(-0.0043334)^3 + 2400\,(-0.0043334)^2 + 241\,(-0.0043334) + 1\right)\Big/\left((-0.0043334 - 1)\,(-0.0043334 + 1)\left((-0.0043334)^2 - 0.0043334 + 1\right)\right)\right)\right)^3 (-1)\tan\left(\frac{17\pi}{542}\right) = 5.61011\times 10^{-19}\,i + 1.12202\times 10^{-18}\,i\sum_{k=1}^{\infty}(-1)^k q^{2k} \quad \text{for } q = (-1)^{17/542}$$

$$\left(-\left(\left(8639\,(-0.0043334)^4 + 8879\,(-0.0043334)^3 + 2400\,(-0.0043334)^2 + 241\,(-0.0043334) + 1\right)\Big/\left((-0.0043334 - 1)\,(-0.0043334 + 1)\left((-0.0043334)^2 - 0.0043334 + 1\right)\right)\right)\right)^3 (-1)\tan\left(\frac{17\pi}{542}\right) = \frac{1.4077\times 10^{-19}\sum_{k=1}^{\infty}\frac{1}{\frac{73152}{73441} - 4k + 4k^2}}{\pi}$$

$$\left(-\left(\left(8639\,(-0.0043334)^4 + 8879\,(-0.0043334)^3 + 2400\,(-0.0043334)^2 + 241\,(-0.0043334) + 1\right)\Big/\left((-0.0043334 - 1)\,(-0.0043334 + 1)\left((-0.0043334)^2 - 0.0043334 + 1\right)\right)\right)\right)^3 (-1)\tan\left(\frac{17\pi}{542}\right) = 5.61011\times 10^{-19}\, i \sum_{k=-\infty}^{\infty} (-1)^k e^{(17\, i\, k\, \pi)/271}\operatorname{sgn}(k)$$

Integral representations

$$\left(-\left(\left(8639\,(-0.0043334)^4 + 8879\,(-0.0043334)^3 + 2400\,(-0.0043334)^2 + 241\,(-0.0043334) + 1\right)\Big/\left((-0.0043334 - 1)\,(-0.0043334 + 1)\left((-0.0043334)^2 - 0.0043334 + 1\right)\right)\right)\right)^3 (-1)\tan\left(\frac{17\pi}{542}\right) = 5.61011\times 10^{-19}\int_0^{\frac{17\pi}{542}}\sec^2(t)\,dt$$

$$\left(-\left(\left(8639\,(-0.0043334)^4 + 8879\,(-0.0043334)^3 + 2400\,(-0.0043334)^2 + 241\,(-0.0043334) + 1\right)\Big/\left((-0.0043334 - 1)\,(-0.0043334 + 1)\left((-0.0043334)^2 - 0.0043334 + 1\right)\right)\right)\right)^3 (-1)\tan\left(\frac{17\pi}{542}\right) = \frac{1.12202\times 10^{-18}}{\pi}\int_0^{\infty}\frac{-1 + t^{17/271}}{-1 + t^2}\,dt$$

Multiple-argument formulas

$$\Big(-\big(\big(8639\,(-0.0043334)^4+8879\,(-0.0043334)^3+2400\,(-0.0043334)^2+241\,(-0.0043334)+1\big)/\big((-0.0043334-1)\,(-0.0043334+1)\,\big((-0.0043334)^2-0.0043334+1\big)\big)\big)\Big)^3\,(-1)\tan\left(\frac{17\pi}{542}\right)=-\frac{1.12202\times 10^{-18}\tan\left(\frac{17\pi}{1084}\right)}{-1+\tan^2\left(\frac{17\pi}{1084}\right)}$$

$$\Big(-\big(\big(8639\,(-0.0043334)^4+8879\,(-0.0043334)^3+2400\,(-0.0043334)^2+241\,(-0.0043334)+1\big)/\big((-0.0043334-1)\,(-0.0043334+1)\,\big((-0.0043334)^2-0.0043334+1\big)\big)\big)\Big)^3\,(-1)\tan\left(\frac{17\pi}{542}\right)=\frac{1.68303\times 10^{-18}\tan\left(\frac{17\pi}{1626}\right)-5.61011\times 10^{-19}\tan^3\left(\frac{17\pi}{1626}\right)}{1-3\tan^2\left(\frac{17\pi}{1626}\right)}$$

$$\Big(-\big(\big(8639\,(-0.0043334)^4+8879\,(-0.0043334)^3+2400\,(-0.0043334)^2+241\,(-0.0043334)+1\big)/\big((-0.0043334-1)\,(-0.0043334+1)\,\big((-0.0043334)^2-0.0043334+1\big)\big)\big)\Big)^3\,(-1)\tan\left(\frac{17\pi}{542}\right)=\frac{5.61011\times 10^{-19}\left(\tan\left(-\frac{525\pi}{542}\right)+\tan(\pi)\right)}{1-\tan\left(-\frac{525\pi}{542}\right)\tan(\pi)}$$

$$\Big(-\big(\big(8639\,(-0.0043334)^4+8879\,(-0.0043334)^3+2400\,(-0.0043334)^2+241\,(-0.0043334)+1\big)/\big((-0.0043334-1)\,(-0.0043334+1)\,\big((-0.0043334)^2-0.0043334+1\big)\big)\big)\Big)^3\,(-1)\tan\left(\frac{17\pi}{542}\right)=\frac{5.61011\times 10^{-19}\,U_{-\frac{525}{542}}(\cos(\pi))\sin(\pi)}{T_{\frac{17}{542}}(\cos(\pi))}$$

$T_n(x)$ is the Chebyshev polynomial of the first kind

$U_n(x)$ is the Chebyshev polynomial of the second kind

And also:

((-(8639 (-0.0043334)^4+8879 (-0.0043334)^3+2400 (-0.0043334)^2+241 (-0.0043334)+1)/(((-0.0043334)-1) ((-0.0043334)+1) ((-0.0043334)^2+(-0.0043334)+1))))^20 (((log^(2/3)(8))/(2^(1/6) log^(5/6)(3))))

where

$$\frac{\log^{2/3}(8)}{\sqrt[6]{2}\,\log^{5/6}(3)} \approx 1.34201004768$$

Input

$$\left(-\left(\left(8639\,(-0.0043334)^4 + 8879\,(-0.0043334)^3 + 2400\,(-0.0043334)^2 + 241\times(-0.0043334) + 1\right)\Big/\left((-0.0043334 - 1)\,(-0.0043334 + 1)\left((-0.0043334)^2 - 0.0043334 + 1\right)\right)\right)\right)^{20} \times \frac{\log^{2/3}(8)}{\sqrt[6]{2}\,\log^{5/6}(3)}$$

log(x) is the natural logarithm

Result

$2.84599\ldots \times 10^{-122}$

$2.84599\ldots*10^{-122}$

$$\Lambda = 3H^{2} = \lambda_P\,(H / h_P)^{2} = \lambda_P\,(l_P / L_H)^{2} = (2.846 \pm 0.076)\;10^{-122}\,m_P^{2}$$

Alternative representations

$$\frac{\left(-\frac{8639\,(-0.0043334)^4+8879\,(-0.0043334)^3+2400\,(-0.0043334)^2+241\,(-0.0043334)+1}{(-0.0043334-1)\,(-0.0043334+1)\left((-0.0043334)^2-0.0043334+1\right)}\right)^{20}\log^{2/3}(8)}{\sqrt[6]{2}\,\log^{5/6}(3)} = \frac{\log_e^{2/3}(8)\left(-\frac{0.0443494-2400\,(-0.0043334)^2-8879\,(-0.0043334)^3-8639\,(-0.0043334)^4}{0.999981\left(0.995667+(-0.0043334)^2\right)}\right)^{20}}{\sqrt[6]{2}\,\log_e^{5/6}(3)}$$

$$\frac{\left(-\frac{8639\,(-0.0043334)^4+8879\,(-0.0043334)^3+2400\,(-0.0043334)^2+241\,(-0.0043334)+1}{(-0.0043334-1)\,(-0.0043334+1)\,\left((-0.0043334)^2-0.0043334+1\right)}\right)^{20}\log^{2/3}(8)}{\sqrt[6]{2}\,\log^{5/6}(3)} = \frac{(\log(a)\log_a(8))^{2/3}\left(-\frac{0.0443494-2400\,(-0.0043334)^2-8879\,(-0.0043334)^3-8639\,(-0.0043334)^4}{0.999981\left(0.995667+(-0.0043334)^2\right)}\right)^{20}}{\sqrt[6]{2}\,(\log(a)\log_a(3))^{5/6}}$$

$$\frac{\left(-\frac{8639\,(-0.0043334)^4+8879\,(-0.0043334)^3+2400\,(-0.0043334)^2+241\,(-0.0043334)+1}{(-0.0043334-1)\,(-0.0043334+1)\,\left((-0.0043334)^2-0.0043334+1\right)}\right)^{20}\log^{2/3}(8)}{\sqrt[6]{2}\,\log^{5/6}(3)} = \frac{(-\mathrm{Li}_1(-7))^{2/3}\left(-\frac{0.0443494-2400\,(-0.0043334)^2-8879\,(-0.0043334)^3-8639\,(-0.0043334)^4}{0.999981\left(0.995667+(-0.0043334)^2\right)}\right)^{20}}{\sqrt[6]{2}\,(-\mathrm{Li}_1(-2))^{5/6}}$$

Series representations

$$\frac{\left(-\frac{8639\,(-0.0043334)^4+8879\,(-0.0043334)^3+2400\,(-0.0043334)^2+241\,(-0.0043334)+1}{(-0.0043334-1)\,(-0.0043334+1)\,\left((-0.0043334)^2-0.0043334+1\right)}\right)^{20}\log^{2/3}(8)}{\sqrt[6]{2}\,\log^{5/6}(3)} = \frac{1.88932\times 10^{-122}\left(\log(7)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{7}\right)^k}{k}\right)^{2/3}}{\left(\log(2)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k}{k}\right)^{5/6}}$$

$$\frac{\left(-\frac{8639\,(-0.0043334)^4+8879\,(-0.0043334)^3+2400\,(-0.0043334)^2+241\,(-0.0043334)+1}{(-0.0043334-1)\,(-0.0043334+1)\,\left((-0.0043334)^2-0.0043334+1\right)}\right)^{20}\log^{2/3}(8)}{\sqrt[6]{2}\,\log^{5/6}(3)} = \frac{1.88932\times 10^{-122}\left(2\,i\,\pi\left\lfloor\frac{\arg(8-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(8-x)^k\,x^{-k}}{k}\right)^{2/3}}{\left(2\,i\,\pi\left\lfloor\frac{\arg(3-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}}{k}\right)^{5/6}} \quad \text{for } x<0$$

$$\frac{\left(-\frac{8639\,(-0.0043334)^4+8879\,(-0.0043334)^3+2400\,(-0.0043334)^2+241\,(-0.0043334)+1}{(-0.0043334-1)\,(-0.0043334+1)\,\left((-0.0043334)^2-0.0043334+1\right)}\right)^{20}\log^{2/3}(8)}{\sqrt[6]{2}\,\log^{5/6}(3)} =$$

$$\frac{1.88932\times 10^{-122}\left(2\,i\,\pi\left\lfloor-\frac{-\pi+\arg\left(\frac{8}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(8-z_0)^k\,z_0^{-k}}{k}\right)^{2/3}}{\left(2\,i\,\pi\left\lfloor-\frac{-\pi+\arg\left(\frac{3}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(3-z_0)^k\,z_0^{-k}}{k}\right)^{5/6}}$$

Integral representations

$$\frac{\left(-\frac{8639\,(-0.0043334)^4+8879\,(-0.0043334)^3+2400\,(-0.0043334)^2+241\,(-0.0043334)+1}{(-0.0043334-1)\,(-0.0043334+1)\,\left((-0.0043334)^2-0.0043334+1\right)}\right)^{20}\log^{2/3}(8)}{\sqrt[6]{2}\,\log^{5/6}(3)} =$$

$$\frac{1.88932\times 10^{-122}\left(\int_1^8\frac{1}{t}\,dt\right)^{2/3}}{\left(\int_1^3\frac{1}{t}\,dt\right)^{5/6}}$$

$$\frac{\left(-\frac{8639\,(-0.0043334)^4+8879\,(-0.0043334)^3+2400\,(-0.0043334)^2+241\,(-0.0043334)+1}{(-0.0043334-1)\,(-0.0043334+1)\,\left((-0.0043334)^2-0.0043334+1\right)}\right)^{20}\log^{2/3}(8)}{\sqrt[6]{2}\,\log^{5/6}(3)} =$$

$$\left(2.12069\times 10^{-122}\,i\,\pi\,\sqrt[6]{\frac{1}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\left(\frac{1}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{7^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^{2/3}\right)\Big/\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\ \text{for } -1<\gamma<0$$

And again:

16*((((-1/((-(8639(-0.0043334)^4+8879(-0.0043334)^3+2400(-0.0043334)^2+241(-0.0043334)+1)/(((-0.0043334)-1)((-0.0043334)+1)((-0.0043334)^2+(-0.0043334)+1))))))^1/3+sqrt2))+((√(10-2√5) -2))⁄((√5-1))

Input

$$16\left(\sqrt[3]{\frac{-1}{-\frac{8639(-0.0043334)^4+8879(-0.0043334)^3+2400(-0.0043334)^2+241\times(-0.0043334)+1}{(-0.0043334-1)(-0.0043334+1)\left((-0.0043334)^2-0.0043334+1\right)}}}+\sqrt{2}\right)+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

Result

1729.0410803696208214501047056674418681261914470525749985409771883
...

1729.04108036962….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = 8^2 * 3^3) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(16*((((-1/((-(8639(-0.0043334)^4+8879(-0.0043334)^3+2400(-0.0043334)^2+241(-0.0043334)+1)/(((-0.0043334)-1)((-0.0043334)+1)((-0.0043334)^2+(-0.0043334)+1))))))^1/3+sqrt2))+(0.284079))^1/15

Input interpretation

$$\left(16\left(\sqrt[3]{\frac{-1}{-\frac{8639(-0.0043334)^4+8879(-0.0043334)^3+2400(-0.0043334)^2+241\times(-0.0043334)+1}{(-0.0043334-1)(-0.0043334+1)\left((-0.0043334)^2-0.0043334+1\right)}}}+\sqrt{2}\right)+0.284079\right)\hat{}\,(1/15)$$

Result

1.6438178324779121928986355485807396465979385513714996699953726748
…

1.643817832477….≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

(1/27((16*((((-1/((-(8639(-0.0043334)^4+8879(-0.0043334)^3+2400(-0.0043334)^2+241(-0.0043334)+1)/(((-0.0043334)-1)((-0.0043334)+1)((-0.0043334)^2+(-0.0043334)+1))))))^1/3+sqrt2))+(0.284079))-1))^2

Input interpretation

$$\left(\frac{1}{27}\left(\left(16\left(\left((-1)/\left(-\left(\left(8639(-0.0043334)^4+8879(-0.0043334)^3+2400(-0.0043334)^2+241\times(-0.0043334)+1\right)/\left((-0.0043334-1)(-0.0043334+1)\left((-0.0043334)^2-0.0043334+1\right)\right)\right)\right)\right)\hat{}(1/3)+\sqrt{2}\right)+0.284079\right)-1\right)\right)^2$$

Result

4096.1947534890127026900371195071146434361981517492889559747258788
…

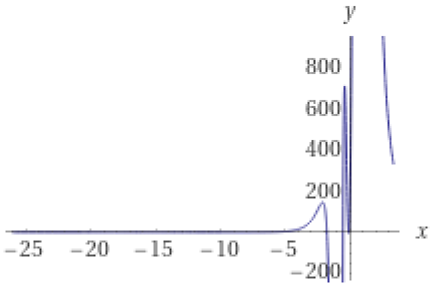
4096.194753489….≈ 4096 = 64^2

derivative(-(8639 x^4 + 8879 x^3 + 2400 x^2 + 241 x + 1)/((x - 1) (x + 1) (x^2 + x + 1)))

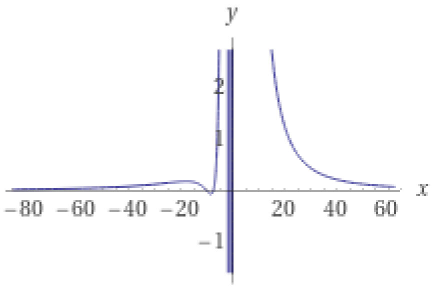
Derivative

$$\frac{d}{dx}\left(-\frac{8639x^4+8879x^3+2400x^2+241x+1}{(x-1)(x+1)\left(x^2+x+1\right)}\right)=\frac{240\left(x^6+20x^5+121x^4+220x^3+121x^2+20x+1\right)}{\left(x^4+x^3-x-1\right)^2}$$

Plots



(x from −26 to 3.3)



(x from −84.6 to 61.9)

Alternate forms

$$\frac{240\,(x\,(x+10)+1)\,(x\,(x\,(x\,(x+10)+20)+10)+1)}{\left(x^4+x^3-x-1\right)^2}$$

$$\frac{240\left(x^2+10\,x+1\right)\left(x^4+10\,x^3+20\,x^2+10\,x+1\right)}{(x-1)^2\,(x+1)^2\left(x^2+x+1\right)^2}$$

$$\frac{x\,(x\,(x\,(x\,(x\,(240\,x+4800)+29\,040)+52\,800)+29\,040)+4800)+240}{x\,(x\,(x\,(x\,(x\,(x\,(x\,(x+2)+1)-2)-4)-2)+1)+2)+1}$$

Partial fraction expansion

$$-\frac{6480\,x}{\left(x^2+x+1\right)^2}-\frac{2160}{x^2+x+1}+\frac{3360}{(x-1)^2}-\frac{960}{(x+1)^2}$$

Expanded forms

$$\frac{240\,x^6}{x^8+2\,x^7+x^6-2\,x^5-4\,x^4-2\,x^3+x^2+2\,x+1}+\frac{4800\,x^5}{x^8+2\,x^7+x^6-2\,x^5-4\,x^4-2\,x^3+x^2+2\,x+1}+\frac{29\,040\,x^4}{x^8+2\,x^7+x^6-2\,x^5-4\,x^4-2\,x^3+x^2+2\,x+1}+\frac{52\,800\,x^3}{x^8+2\,x^7+x^6-2\,x^5-4\,x^4-2\,x^3+x^2+2\,x+1}+\frac{29\,040\,x^2}{x^8+2\,x^7+x^6-2\,x^5-4\,x^4-2\,x^3+x^2+2\,x+1}+\frac{4800\,x}{x^8+2\,x^7+x^6-2\,x^5-4\,x^4-2\,x^3+x^2+2\,x+1}+\frac{240}{x^8+2\,x^7+x^6-2\,x^5-4\,x^4-2\,x^3+x^2+2\,x+1}$$

$$\frac{29\,040\,x^4}{(x^4+x^3-x-1)^2}+\frac{52\,800\,x^3}{(x^4+x^3-x-1)^2}+\frac{4800\,x}{(x^4+x^3-x-1)^2}+\frac{240}{(x^4+x^3-x-1)^2}+\frac{240\,x^6}{(x^4+x^3-x-1)^2}+\frac{4800\,x^5}{(x^4+x^3-x-1)^2}+\frac{29\,040\,x^2}{(x^4+x^3-x-1)^2}$$

$$\frac{240\,x^6+4800\,x^5+29\,040\,x^4+52\,800\,x^3+29\,040\,x^2+4800\,x+240}{x^8+2\,x^7+x^6-2\,x^5-4\,x^4-2\,x^3+x^2+2\,x+1}$$

Roots

$$x=-5-2\sqrt{6}$$

$$x=2\sqrt{6}-5$$

$$x=\frac{1}{2}\left(-5+\sqrt{7}+\sqrt{28-10\sqrt{7}}\right)$$

$$x = \frac{1}{2}\left(-5 + \sqrt{7} - \sqrt{28 - 10\sqrt{7}}\right)$$

$$x = \frac{1}{2}\left(-5 - \sqrt{7} - \sqrt{28 + 10\sqrt{7}}\right)$$

Roots

$x \approx -9.8990$

$x \approx -0.10102$

$x \approx -0.55614$

$x \approx -1.7981$

$x \approx -7.5126$

Properties as a real function

Domain

$\{x \in \mathbb{R} : x \neq -1 \text{ and } x \neq 1\}$

Range

$\mathbb{R}$ (all real numbers)

Surjectivity

surjective onto $\mathbb{R}$

$\mathbb{R}$ is the set of real numbers

Series expansion at x=0

$$240 + 4320\,x + 20\,160\,x^2 + 8640\,x^3 + 1200\,x^4 + O(x^5)$$

(Taylor series)

Series expansion at x=∞

$$\frac{240}{x^2} + \frac{4320}{x^3} + \frac{20\,160}{x^4} + \frac{8640}{x^5} + O\left(\left(\frac{1}{x}\right)^6\right)$$

(Laurent series)

Indefinite integral

$$\int \frac{240\,(1 + 20\,x + 121\,x^2 + 220\,x^3 + 121\,x^4 + 20\,x^5 + x^6)}{(-1 - x + x^3 + x^4)^2}\,dx =$$
$$-\frac{240\,(x^3 + 10\,x^2 + 37\,x + 36)}{x^4 + x^3 - x - 1} + \text{constant}$$

Local maxima

$$\max\left\{\frac{240\,(x^6 + 20\,x^5 + 121\,x^4 + 220\,x^3 + 121\,x^2 + 20\,x + 1)}{(x^4 + x^3 - x - 1)^2}\right\} \approx 0.18743$$

at $x \approx -17.900$

$$\max\left\{\frac{240\,(x^6 + 20\,x^5 + 121\,x^4 + 220\,x^3 + 121\,x^2 + 20\,x + 1)}{(x^4 + x^3 - x - 1)^2}\right\} \approx 146.29$$

at $x \approx -2.1306$

Local minima

$$\min\left\{\frac{240\left(x^6+20\,x^5+121\,x^4+220\,x^3+121\,x^2+20\,x+1\right)}{\left(x^4+x^3-x-1\right)^2}\right\}\approx -0.058758$$

$$\text{at } x\approx -8.3981$$

$$\min\left\{\frac{240\left(x^6+20\,x^5+121\,x^4+220\,x^3+121\,x^2+20\,x+1\right)}{\left(x^4+x^3-x-1\right)^2}\right\}\approx -4.2183$$

$$\text{at } x\approx -0.11695$$

Limit

$$\lim_{x\to\pm\infty}\frac{240\left(1+20\,x+121\,x^2+220\,x^3+121\,x^4+20\,x^5+x^6\right)}{\left(-1-x+x^3+x^4\right)^2}=0$$

From

$$\frac{d}{dx}\left(-\frac{8639\,x^4+8879\,x^3+2400\,x^2+241\,x+1}{(x-1)\,(x+1)\left(x^2+x+1\right)}\right)=\frac{240\left(x^6+20\,x^5+121\,x^4+220\,x^3+121\,x^2+20\,x+1\right)}{\left(x^4+x^3-x-1\right)^2}$$

For x = - 0.10102

1/((240(1+20(-0.10102)+121(-0.10102)^2+220(-0.10102)^3+121(-0.10102)^4+20(-0.10102)^5+(-0.10102)^6))/(-1 - (-0.10102)+(-0.10102)^3+(-0.10102)^4)^2)^34 * ((-1 + e + π - sqrt(1 + π)))

where $-1+e+\pi-\sqrt{1+\pi}\approx 2.8247841514$

Input

$$1\Big/\left(\left(240\left(1+20\times(-0.10102)+121\,(-0.10102)^2+220\,(-0.10102)^3+121\,(-0.10102)^4+20\,(-0.10102)^5+(-0.10102)^6\right)\right)\Big/\left(-1--0.10102+(-0.10102)^3+(-0.10102)^4\right)^2\right)^{34}\left(-1+e+\pi-\sqrt{1+\pi}\right)$$

Result

$3.5160012104208520240796920924414293807044538282851857606057\ldots \times 10^{121}$

$0.351600121\ldots{*}10^{122}$

$$\Lambda_Q = 3H_Q^{\,2} = \lambda_P\,(h_P\,/\,H\,)^2 = \lambda_P\,(L_H/\,l_P)^2 = (0.3516\ \pm\ 0.094)\ 10^{122} m_P^{\,2}$$

Series representations

$$\left(-1+e+\pi-\sqrt{1+\pi}\right)/\left(\left(240\left(1+20\,(-0.10102)+121\,(-0.10102)^2+220\,(-0.10102)^3+121\,(-0.10102)^4+20\,(-0.10102)^5+(-0.10102)^6\right)\right)/\left(-1--0.10102+(-0.10102)^3+(-0.10102)^4\right)^2\right)^{34} = -1.2447\times10^{121}+1.2447\times10^{121}\,e+1.2447\times10^{121}\,\pi-1.2447\times10^{121}\sqrt{\pi}\sum_{k=0}^{\infty}\pi^{-k}\binom{\frac{1}{2}}{k}$$

$$\left(-1+e+\pi-\sqrt{1+\pi}\right)/\left(\left(240\left(1+20\,(-0.10102)+121\,(-0.10102)^2+220\,(-0.10102)^3+121\,(-0.10102)^4+20\,(-0.10102)^5+(-0.10102)^6\right)\right)/\left(-1--0.10102+(-0.10102)^3+(-0.10102)^4\right)^2\right)^{34} = -1.2447\times10^{121}+1.2447\times10^{121}\,e+1.2447\times10^{121}\,\pi-1.2447\times10^{121}\sqrt{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k\,\pi^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

$$\left(-1+e+\pi-\sqrt{1+\pi}\right)/\left(\left(240\left(1+20\,(-0.10102)+121\,(-0.10102)^2+220\,(-0.10102)^3+121\,(-0.10102)^4+20\,(-0.10102)^5+(-0.10102)^6\right)\right)/\left(-1--0.10102+(-0.10102)^3+(-0.10102)^4\right)^2\right)^{34} = -1.2447\times10^{121}+1.2447\times10^{121}\,e+1.2447\times10^{121}\,\pi-\frac{6.22349\times10^{120}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\pi^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}$$

integrate(-(8639 x^4 + 8879 x^3 + 2400 x^2 + 241 x + 1)/((x - 1) (x + 1) (x^2 + x + 1)))

Indefinite integral

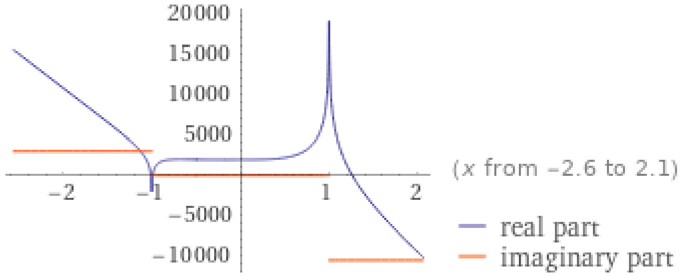
$$\int -\frac{8639\,x^4 + 8879\,x^3 + 2400\,x^2 + 241\,x + 1}{(x-1)\,(x+1)\,(x^2+x+1)}\,dx = 1080\log(x^2+x+1) - 8639\,x - 3360\log(1-x) + 960\log(x+1) + 2160\sqrt{3}\tan^{-1}\left(\frac{2\,x+1}{\sqrt{3}}\right) + \text{constant}$$

(assuming a complex-valued logarithm)

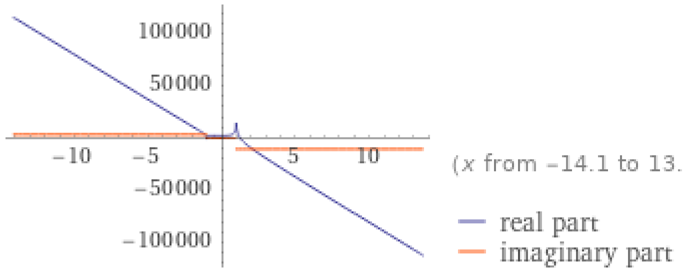
$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Plots of the integral

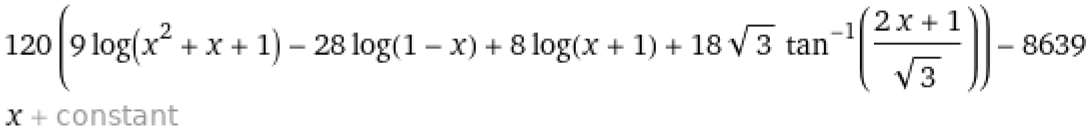


(x from −2.6 to 2.1)

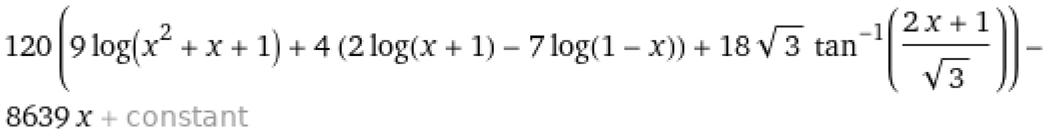


(x from −14.1 to 13.6)

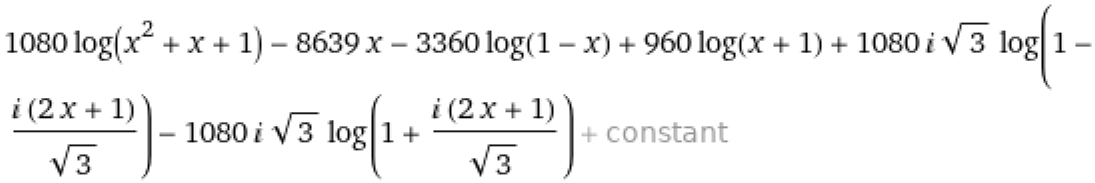
Alternate forms of the integral



$$120\left(9\log\left(x^2+x+1\right)-28\log(1-x)+8\log(x+1)+18\sqrt{3}\,\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)-8639\,x+\text{constant}$$

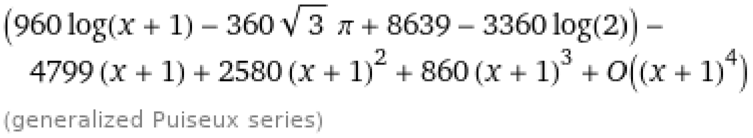


$$120\left(9\log\left(x^2+x+1\right)+4\,(2\log(x+1)-7\log(1-x))+18\sqrt{3}\,\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)-8639\,x+\text{constant}$$



$$1080\log\left(x^2+x+1\right)-8639\,x-3360\log(1-x)+960\log(x+1)+1080\,i\sqrt{3}\,\log\left(1-\frac{i\,(2x+1)}{\sqrt{3}}\right)-1080\,i\sqrt{3}\,\log\left(1+\frac{i\,(2x+1)}{\sqrt{3}}\right)+\text{constant}$$

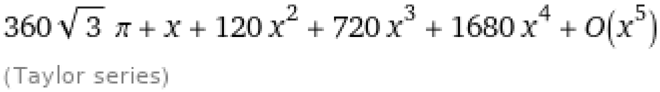
Series expansion of the integral at x=-1



$$\left(960\log(x+1)-360\sqrt{3}\,\pi+8639-3360\log(2)\right)-4799\,(x+1)+2580\,(x+1)^2+860\,(x+1)^3+O\left((x+1)^4\right)$$

(generalized Puiseux series)

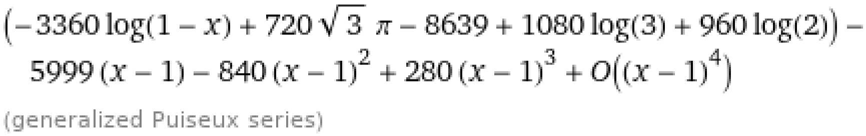
Series expansion of the integral at x=0



$$360\sqrt{3}\,\pi+x+120\,x^2+720\,x^3+1680\,x^4+O\left(x^5\right)$$

(Taylor series)

Series expansion of the integral at x=1



$$\left(-3360\log(1-x)+720\sqrt{3}\,\pi-8639+1080\log(3)+960\log(2)\right)-5999\,(x-1)-840\,(x-1)^2+280\,(x-1)^3+O\left((x-1)^4\right)$$

(generalized Puiseux series)

Series expansion of the integral at x=-(-1)$^{1/3}$

$$-2160\sqrt{3}\,\pi\left\lfloor\frac{3}{4}-\frac{\arg\left(x+\sqrt[3]{-1}\right)}{2\pi}\right\rfloor+\left(\left(1080\,i\sqrt{3}\,\log\left(x+\sqrt[3]{-1}\right)+1080\log\left(-i\sqrt{3}\left(x+\sqrt[3]{-1}\right)\right)-3360\log\left(3+i\sqrt{3}\right)-540\,i\sqrt{3}\,\log(3)+3360\log(2)+540\sqrt{3}\,\pi-320\,i\,\pi+8639\sqrt[3]{-1}\right)-\frac{\left(15\,357\,i+6239\sqrt{3}\right)\left(x+\sqrt[3]{-1}\right)}{3\,i+\sqrt{3}}+\frac{1400\left(-i+\sqrt{3}\right)\left(x+\sqrt[3]{-1}\right)^{2}}{i+\sqrt{3}}+\frac{40}{9}\left(-99-37\,i\sqrt{3}\right)\left(x+\sqrt[3]{-1}\right)^{3}+O\left(\left(x+\sqrt[3]{-1}\right)^{4}\right)\right)$$

Series expansion of the integral at x=(-1)$^{2/3}$

$$2160\sqrt{3}\,\pi\left\lfloor\frac{\pi-2\arg\left(x-(-1)^{2/3}\right)}{4\pi}\right\rfloor+\left(\left(-1080\,i\sqrt{3}\,\log\left(x-(-1)^{2/3}\right)+1080\log\left(i\sqrt{3}\left(x-(-1)^{2/3}\right)\right)-3360\log\left(3-i\sqrt{3}\right)+540\,i\sqrt{3}\,\log(3)+3360\log(2)+540\sqrt{3}\,\pi+320\,i\,\pi-8639\,(-1)^{2/3}\right)+\frac{\left(15\,357\,i-6239\sqrt{3}\right)\left(x-(-1)^{2/3}\right)}{-3\,i+\sqrt{3}}+\frac{1400\left(i+\sqrt{3}\right)\left(x-(-1)^{2/3}\right)^{2}}{-i+\sqrt{3}}+\frac{40}{9}\left(-99+37\,i\sqrt{3}\right)\left(x-(-1)^{2/3}\right)^{3}+O\left(\left(x-(-1)^{2/3}\right)^{4}\right)\right)$$

Series expansion of the integral at x = 1/2 (-1 - i sqrt(3))

$$\Bigg(\frac{1}{2}\Big(2160\,i\sqrt{3}\,\log\Big(x+\frac{1}{2}(1+i\sqrt{3})\Big)+2160\log\Big(-i\sqrt{3}\,\Big(x+\frac{1}{2}(1+i\sqrt{3})\Big)\Big)-6720\log(3+i\sqrt{3})-1080\,i\sqrt{3}\,\log(3)+6720\log(2)+1080\sqrt{3}\,\pi-640\,i\,\pi+8639\,i\sqrt{3}+8639\Big)+\frac{2\,i\,(15357\,i+6239\sqrt{3})\,\big(x-\frac{1}{2}(-1-i\sqrt{3})\big)}{(i+\sqrt{3})(-3\,i+\sqrt{3})}-\frac{33600\,i\,(-i+\sqrt{3})\,\big(x-\frac{1}{2}(-1-i\sqrt{3})\big)^2}{(i+\sqrt{3})^2(-3\,i+\sqrt{3})^2}+\frac{2560\,i\,(37\,i+33\sqrt{3})\,\big(x-\frac{1}{2}(-1-i\sqrt{3})\big)^3}{(i+\sqrt{3})^3(-3\,i+\sqrt{3})^3}+O\Big(\Big(x-\frac{1}{2}(-1-i\sqrt{3})\Big)^4\Big)\Bigg)-2160\sqrt{3}\,\pi\left\lfloor\frac{-\arg(2x+i\sqrt{3}+1)-\arg\left(-\frac{2\sqrt{3}\,x+\sqrt{3}+3\,i}{-2\,i\,x+\sqrt{3}-i}\right)+\pi}{2\pi}\right\rfloor$$

From:

$$\int-\frac{8639x^4+8879x^3+2400x^2+241x+1}{(x-1)(x+1)(x^2+x+1)}\,dx=1080\log(x^2+x+1)-8639x-3360\log(1-x)+960\log(x+1)+2160\sqrt{3}\,\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\text{constant}$$

(assuming a complex-valued logarithm)

1/9((1080 log(2.1^2 + 2.1 + 1) - 8639 2.1 - 3360 log(1 - 2.1) + 960 log(2.1 + 1) + 2160 sqrt(3) tan^(-1)((2 2.1 + 1)/sqrt(3))))-Pi*i-(24*4)*i

Input

$$\frac{1}{9}\left(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+960\log(2.1+1)+2160\sqrt{3}\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\right)-\pi i-(24\times 4)i$$

$-1169.41\ldots -$
$1272.00\ldots i$

(result in radians)

Polar coordinates

$r = 1727.86$ (radius), $\theta = -2.3142$ (angle)

$1727.86 \approx 1728$

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Polar forms

$1727.86(\cos(-2.3142)+i\sin(-2.3142))$

$1727.86\,e^{-2.3142\,i}$

Alternative representations

$$\frac{1}{9}\left(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+\right.$$
$$\left.960\log(2.1+1)+2160\sqrt{3}\,\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\right)-i\,\pi-i\,24\times 4=$$
$$-96\,i-i\,\pi+\frac{1}{9}\left(-18\,141.9-3360\log(-1.1)+960\log(3.1)+\right.$$
$$\left.1080\log\left(3.1+2.1^2\right)+2160\tan^{-1}\left(1,\frac{5.2}{\sqrt{3}}\right)\sqrt{3}\right)$$

$$\frac{1}{9}\left(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+\right.$$
$$\left.960\log(2.1+1)+2160\sqrt{3}\,\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\right)-i\,\pi-i\,24\times 4=$$
$$-96\,i-i\,\pi+\frac{1}{9}\left(-18\,141.9-3360\log(-1.1)+960\log(3.1)+\right.$$
$$\left.1080\log\left(3.1+2.1^2\right)+1080\,i\left(\log\left(1-\frac{5.2\,i}{\sqrt{3}}\right)-\log\left(1+\frac{5.2\,i}{\sqrt{3}}\right)\right)\sqrt{3}\right)$$

$$\frac{1}{9}\left(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+\right.$$
$$\left.960\log(2.1+1)+2160\sqrt{3}\,\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\right)-i\,\pi-i\,24\times 4=$$
$$-96\,i-i\,\pi+\frac{1}{9}\left(-18\,141.9-3360\log(a)\log_a(-1.1)+960\log(a)\log_a(3.1)+\right.$$
$$\left.1080\log(a)\log_a\left(3.1+2.1^2\right)+2160\tan^{-1}\left(\frac{5.2}{\sqrt{3}}\right)\sqrt{3}\right)$$

Series representations

$$\frac{1}{9}\left(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+960\log(2.1+1)+2160\sqrt{3}\,\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\right)-i\pi-i\,24\times 4=$$

$$-\left(2015.77+96i+i\pi+373.333\log(-2.1)-106.667\log(2.1)-120\log(6.51)-\sum_{k=1}^{\infty}-\frac{40\left(8(-13.671)^k+9(-4.41)^k-28\times 13.671^k\right)e^{-3.35721k}}{3k}-240\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{1}{k_1!\,(1+2k_2)}(-1)^{k_1+k_2}\,5^{-k_2}\times 10.4^{1+2k_2}\,(3-x)^{k_1}\,x^{-k_1}\,F_{1+2k_2}\left(-\frac{1}{2}\right)_{k_1}\left(\frac{1}{\sqrt{3}\left(1+\sqrt{1+\frac{21.632}{\sqrt{3}^{\,2}}}\right)}\right)^{1+2k_2}\right)\quad \text{for } (x\in\mathbb{R}\text{ and } x<0)$$

$$\frac{1}{9}\bigg(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+$$

$$960\log(2.1+1)+2160\sqrt{3}\,\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\bigg)-i\,\pi-i\,24\times 4=$$

$$-\Bigg(2015.77+96\,i+i\,\pi+373.333\log(-2.1)-106.667\log(2.1)-120\log(6.51)-$$

$$\sum_{k=1}^{\infty}\frac{-120\,(-1)^k\,e^{-1.87334\,k}+\left(373.333-106.667\,(-1)^k\right)e^{-0.741937\,k}}{k}-$$

$$240\left(\frac{1}{z_0}\right)^{1/2\,\lfloor\arg(3-z_0)/(2\,\pi)\rfloor}\,z_0^{1/2+1/2\,\lfloor\arg(3-z_0)/(2\,\pi)\rfloor}$$

$$\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{1}{k_1!\,(1+2\,k_2)}\,(-1)^{k_1+k_2}\,5^{-k_2}\times 10.4^{1+2\,k_2}\,F_{1+2\,k_2}$$

$$\left(-\frac{1}{2}\right)_{k_1}\left(\frac{1}{\sqrt{3}+\sqrt{3}\,\sqrt{1+\frac{21.632}{\sqrt{3}^{\,2}}}}\right)^{1+2\,k_2}(3-z_0)^{k_1}\,z_0^{-k_1}\Bigg)$$

$$\frac{1}{9}\left(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+960\log(2.1+1)+2160\sqrt{3}\,\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\right)-i\pi-i\,24\times 4 =$$

$$-\left(2015.77+96i+i\pi+373.333\log(-2.1)-106.667\log(2.1)-120\log(6.51)-\sum_{k=1}^{\infty}-\frac{40\,e^{-2.61528k}\left(9(-1)^k e^{0.741937k}+4\left(-7+2(-1)^k\right)e^{1.87334k}\right)}{3k}-\sum_{k=0}^{\infty}\frac{1}{k!}\,240(-1)^k(3-x)^k x^{-k}\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\left(\tan^{-1}(x)+\pi\left\lfloor\frac{\arg\left(-x\mathcal{A}+\frac{5.2\mathcal{A}}{\sqrt{3}}\right)}{2\pi}\right\rfloor\right)\left(-\frac{1}{2}\right)_k\sqrt{x}-120\mathcal{A}\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k_1=0}^{\infty}\sum_{k_2=1}^{\infty}\frac{1}{k_1!\,k_2}(-1)^{k_1}(3-x)^{k_1}x^{-k_1}\left(-(-x-\mathcal{A})^{-k_2}+(-x+\mathcal{A})^{-k_2}\right)\left(-\frac{1}{2}\right)_{k_1}\left(-x+\frac{5.2}{\sqrt{3}}\right)^{k_2}\right)$$

for ($ix \in \mathbb{R}$ and $ix < -1$ and $x \in \mathbb{R}$ and $x < 0$)

Integral representations

$$\frac{1}{9}\left(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+960\log(2.1+1)+2160\sqrt{3}\,\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\right)-i\pi-i\,24\times 4 =$$

$$-2015.77-96i-i\pi+1248\int_0^1\frac{1}{1+\frac{27.04t^2}{\sqrt{3}^2}}\,dt-373.333\log(-1.1)+106.667\log(3.1)+120\log(7.51)$$

$$\frac{1}{9}\left(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+960\log(2.1+1)+2160\sqrt{3}\,\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\right)-i\pi-i\,24\times 4=-2015.77-96i-i\pi-\frac{312\mathcal{A}}{\pi^{3/2}}\int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\left(1+\frac{27.04}{\sqrt{3}^2}\right)^{-s}ds-373.333\log(-1.1)+106.667\log(3.1)+120\log(7.51)\quad\text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{1}{9}\left(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+960\log(2.1+1)+2160\sqrt{3}\,\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\right)-i\pi-i\,24\times 4=-2015.77-96i-i\pi+\frac{312}{\pi\mathcal{A}}\int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma}\frac{e^{-3.29732\,s}\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)\left(\frac{1}{\sqrt{3}^2}\right)^{-s}}{\Gamma\left(\frac{3}{2}-s\right)}ds-373.333\log(-1.1)+106.667\log(3.1)+120\log(7.51)\quad\text{for } 0<\gamma<\frac{1}{2}$$

$\Gamma(x)$ is the gamma function

Continued fraction representations

$$\frac{1}{9}\left(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+960\log(2.1+1)+2160\sqrt{3}\,\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\right)-i\pi-i\,24\times 4=-2015.77-i(96+\pi)-\frac{1120\log(-1.1)}{3}+\frac{320\log(3.1)}{3}+120\log(7.51)+\frac{1248}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{27.04k^2}{\sqrt{3}^2}}{1+2k}}=(-1688.72-1272.\,i)+\frac{1248}{1+\frac{9.01333}{3+\frac{36.0533}{5+\frac{81.12}{7+\frac{144.213}{9+\dots}}}}}$$

$$\frac{1}{9}\Big(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+$$

$$960\log(2.1+1)+2160\sqrt{3}\,\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\Big)-i\,\pi-i\,24\times 4=$$

$$-767.767-i\,(96+\pi)-\frac{1120\log(-1.1)}{3}+\frac{320\log(3.1)}{3}+$$

$$120\log(7.51)-\frac{33\,745.9}{\left(3+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{27.04\left(1+(-1)^{1+k}+k\right)^2}{\sqrt{3}^{\,2}}}{3+2k}\right)\sqrt{3}^{\,2}}=$$

$$(-440.718-1272.\,i)-\cfrac{11\,248.6}{3+\cfrac{81.12}{5+\cfrac{36.0533}{7+\cfrac{225.333}{9+\cfrac{144.213}{11+\ldots}}}}}$$

$$\frac{1}{9}\Big(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+$$

$$960\log(2.1+1)+2160\sqrt{3}\,\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\Big)-i\,\pi-i\,24\times 4=$$

$$-2015.77-96\,i-i\,\pi-\frac{1120\log(-1.1)}{3}+\frac{320\log(3.1)}{3}+$$

$$120\log(7.51)+\frac{1248\left(3+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-6}{6}\right)}{\left(1+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{27.04\,k^2}{\sqrt{3}^{\,2}}}{1+2k}\right)\sqrt{3}}=$$

$$(-1688.72-1272.\,i)+\frac{720.533\left(3+-\cfrac{6}{6-\cfrac{6}{6-\cfrac{6}{6-\cfrac{6}{6+\ldots}}}}\right)}{1+\cfrac{9.01333}{3+\cfrac{36.0533}{5+\cfrac{81.12}{7+\cfrac{144.213}{9+\ldots}}}}}$$

$\mathop{\mathrm{K}}_{k=k_1}^{k_2} a_k/b_k$ is a continued fraction

((1/9((1080 log(2.1^2 + 2.1 + 1) - 8639 2.1 - 3360 log(1 - 2.1) + 960 log(2.1 + 1) + 2160 sqrt(3) tan^(-1)((2 2.1 + 1)/sqrt(3))))-Pi*i-(24*4)*i))^1/15

Input

$$\left(\frac{1}{9}\left(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+960\log(2.1+1)+2160\sqrt{3}\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\right)-\pi i-(24\times 4)i\right)\text{\textasciicircum}(1/15)$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

i is the imaginary unit

Result

$1.624219\ldots - 0.2525914\ldots i$

(result in radians)

Polar coordinates

$r = 1.64374$ (radius), $\theta = -0.15428$ (angle)

1.64374….≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

(1/27(1/9((1080 log(2.1^2 + 2.1 + 1) - 8639 2.1 - 3360 log(1 - 2.1) + 960 log(2.1 + 1) + 2160 sqrt(3) tan^(-1)((2 2.1 + 1)/sqrt(3))))-Pi*i-(24*4)*i))^2+Φ*i

Input

$$\left(\frac{1}{27}\left(\frac{1}{9}\left(1080\log\left(2.1^2+2.1+1\right)-8639\times 2.1-3360\log(1-2.1)+960\log(2.1+1)+2160\sqrt{3}\tan^{-1}\left(\frac{2\times 2.1+1}{\sqrt{3}}\right)\right)-\pi i-(24\times 4)i\right)\right)^2+\Phi i$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

i is the imaginary unit

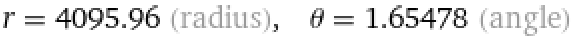
Φ is the golden ratio conjugate

Result

$-343.592\ldots + 4081.52\ldots i$

(result in radians)

Polar coordinates



$r = 4095.96$ (radius), $\theta = 1.65478$ (angle)

4095.96…..≈ 4096 = 64^2

Mathematical connections with some sectors of String Theory

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

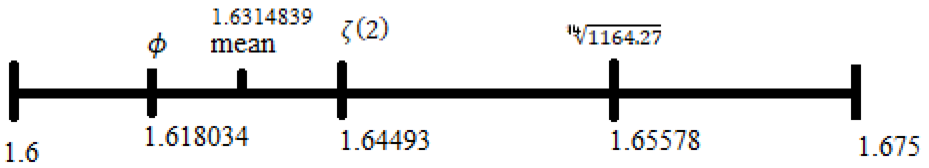
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

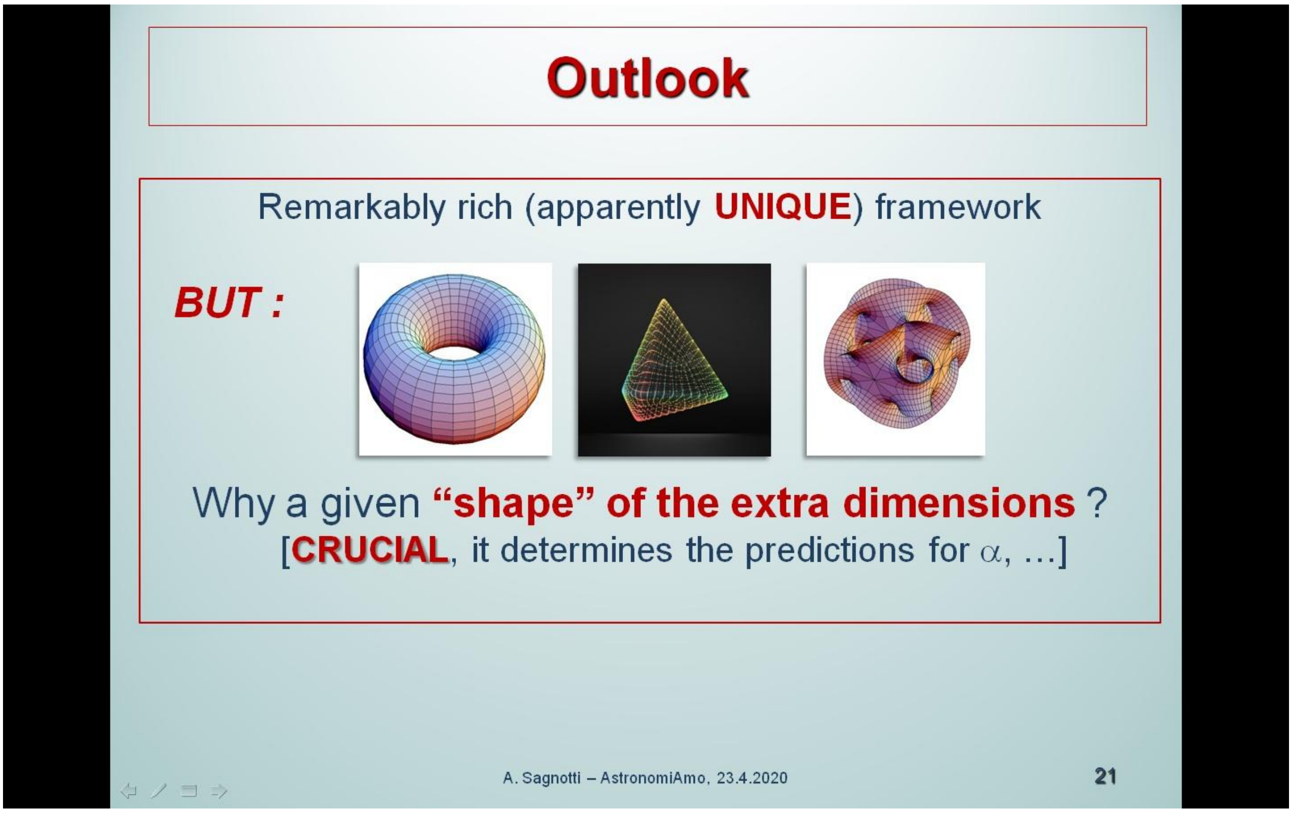
“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

We have that:



Outlook

Remarkably rich (apparently **UNIQUE**) framework

BUT :

Why a given **“shape” of the extra dimensions** ?
[**CRUCIAL**, it determines the predictions for α, …]

A. Sagnotti – AstronomiAmo, 23.4.2020

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From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: “why a given shape of the extra dimensions? Crucial, it determines the predictions for α”.

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and $\zeta(2)$, (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Appoximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We obtain, in certain cases, the following connections:

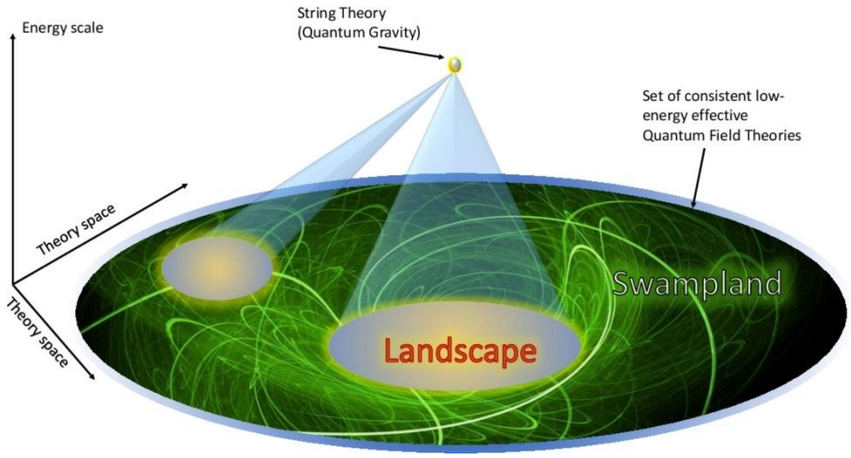
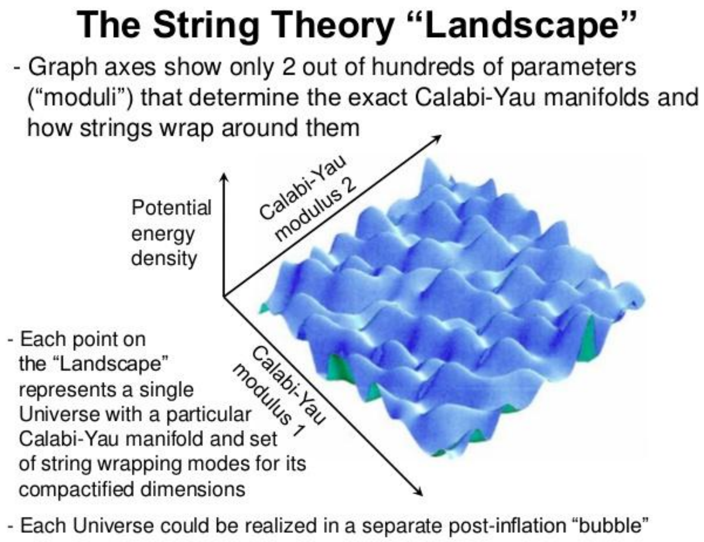


Fig. 1



The String Theory "Landscape"

- Graph axes show only 2 out of hundreds of parameters ("moduli") that determine the exact Calabi-Yau manifolds and how strings wrap around them

- Each point on the "Landscape" represents a single Universe with a particular Calabi-Yau manifold and set of string wrapping modes for its compactified dimensions

- Each Universe could be realized in a separate post-inflation "bubble"

Fig. 2

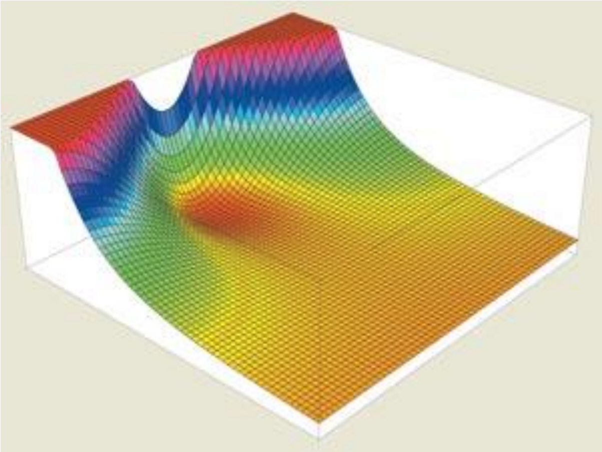


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

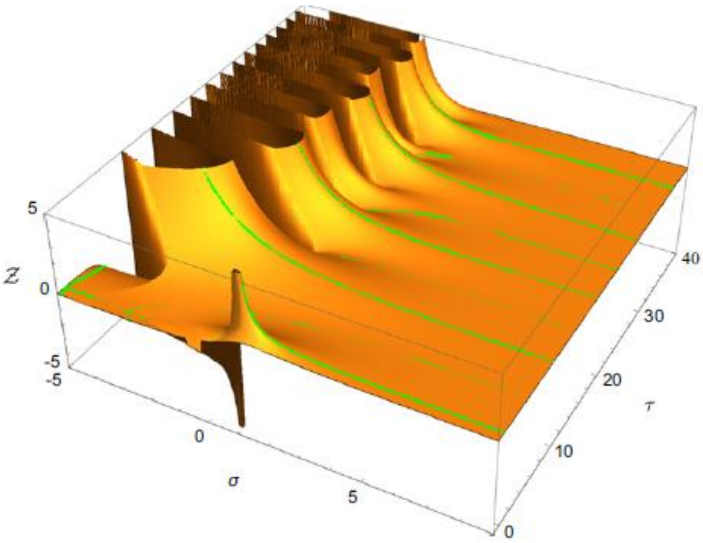


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + \mathrm{i}\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k \, \frac{\pi}{\ln 2},$$

with $k = \ldots, -2, -1, 0, 1, 2, \ldots.$

we obtain:

2Pi/(ln(2))

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.0647202836543876192553658914333336203437229354475911683720330958
…

9.06472028365….

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\,i\,\pi\left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\,i\,\pi\left\lfloor \frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\,dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4\,i\,\pi^2}{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\,\pi\log(2)\right)$$

log(x) is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

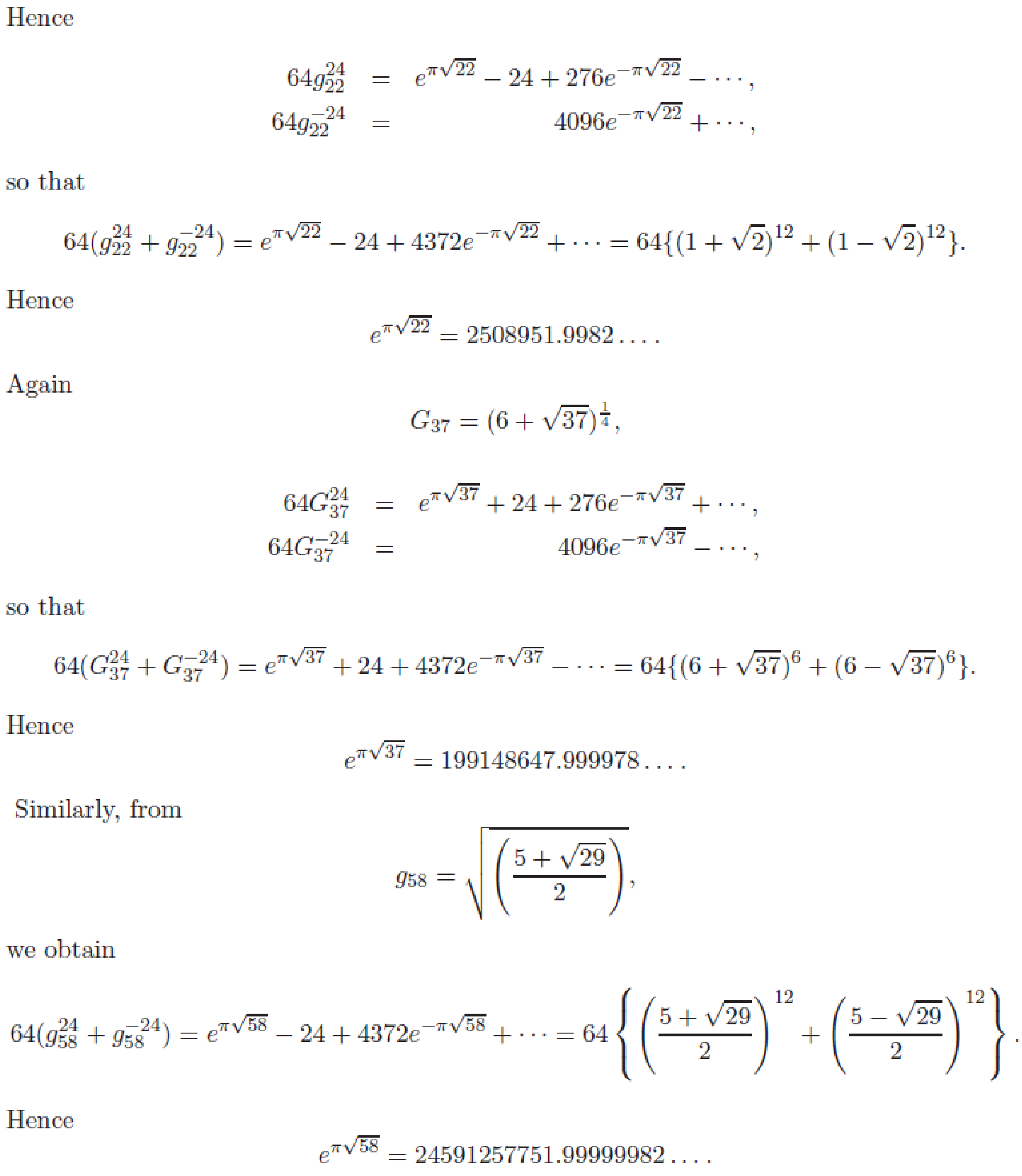
1.6449340668482264364724151666460251892189499012067984377355582293
…

1.6449340668…. = ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

From:

Modular equations and approximations to π - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:



Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))⁄((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023
…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256x^8 + 95744x^7 - 3248750080x^6 - 914210725504x^5 + 15498355554921184x^4 + 2911478392539914656x^3 - 32941144911224677091680x^2 - 3092528914069760354714456x + 26320050609744039027169013041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}+108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}-27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Bigg/\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\right)$$

$$27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}+108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}+2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}-27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(10-2\sqrt{5}-z_0\right)^{k}z_0^{-k}}{k!}\right)\Bigg/\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(5-z_0)^{k}z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))⁄((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023

…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$\begin{aligned}&256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - \\ &\quad 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + \\ &\quad 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - \\ &\quad 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041\end{aligned}$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$\begin{aligned}&27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \\ &\quad \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + \right. \\ &\qquad 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - \\ &\qquad \left. 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\end{aligned}$$

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

$1.64381856858\ldots . \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\boxed{\begin{array}{l}\text{root of }\ 256\,x^8+95\,744\,x^7-3\,248\,750\,080\,x^6-914\,210\,725\,504\,x^5+\\ \quad 15\,498\,355\,554\,921\,184\,x^4+2\,911\,478\,392\,539\,914\,656\,x^3-\\ \quad 32\,941\,144\,911\,224\,677\,091\,680\,x^2-3\,092\,528\,914\,069\,760\,354\,714\,456\,x+\\ \quad 26\,320\,050\,609\,744\,039\,027\,169\,013\,041\ \text{ near }\ x=1729.05\end{array}}}$$

Minimal polynomial

$$\begin{array}{l}256\,x^{120}+95\,744\,x^{105}-3\,248\,750\,080\,x^{90}-\\ \quad 914\,210\,725\,504\,x^{75}+15\,498\,355\,554\,921\,184\,x^{60}+\\ \quad 2\,911\,478\,392\,539\,914\,656\,x^{45}-32\,941\,144\,911\,224\,677\,091\,680\,x^{30}-\\ \quad 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15}+26\,320\,050\,609\,744\,039\,027\,169\,013\,041\end{array}$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right)+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4}+\frac{29\sqrt{5}}{4}+54\sqrt{1093}-\frac{27}{8}\sqrt{10-2\sqrt{5}}-\frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^{0}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.64382\ \text{(real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.50170+0.6686\,i$$

$$e^{(4 i \pi)/15} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)} \approx 1.0999 + 1.2216\, i$$

$$e^{(2 i \pi)/5} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)} \approx 0.5080 + 1.5634\, i$$

$$e^{(8 i \pi)/15} \sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)} \approx -0.17183 + 1.63481\, i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi} = \frac{1}{\sqrt[15]{2}} \left(\left(\left(\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 2\phi\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)\right)\right) \text{^}\,(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi} =$$

$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.$$

$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$

$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$

$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\text{^}(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi} =$$

$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.$$

$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$

$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$

$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$

$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\text{^}(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Integral representation

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\,\pi\,i)\,\Gamma(-a)}$$ for ($0 < \gamma < -\mathrm{Re}(a)$ and $|\arg(z)| < \pi$)

From:

An Update on Brane Supersymmetry Breaking
J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = - \frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)\,C + 2\beta_E^{(p)}\,\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)\,C + 2\beta_E^{(p)}\,\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)\,C + 2\beta_E^{(p)}\,\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)\,C + 2\beta_E^{(p)}\,\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p, C, β_E and ϕ correspond to the exponents of e (i.e. of exp). Thence we obtain for p = 5 and $\beta_E = 1/2$:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18))   we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.6272016226072509292942156739117979541838581136954016\ldots \times 10^{-6}$$

$1.6272016\ldots * 10^{-6}$

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

$e^{-\pi\sqrt{18}}$ = 1.6272016… * 10^-6

from which:

$\frac{1}{4096} e^{-6C+\phi}$ = 1.6272016… * 10^-6

0.000244140625 $e^{-6C+\phi}$ = $e^{-\pi\sqrt{18}}$ = 1.6272016… * 10^-6

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625}\, e^{-6C+\phi} = \frac{1}{0.000244140625}\, e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

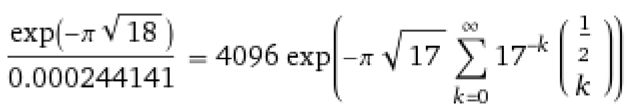
$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:



$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\, \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}} \times \frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

log(0.00666501784619)

Result:

–5.010882647757…

-5.010882647757…

Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2 i \pi \left\lfloor \frac{\arg(0.006665017846190000 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2 \pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2 \pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$\phi = -5.010882647757 + 6 = \mathbf{0.989117352243} = \boldsymbol{\phi}$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.9874428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1} = 1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\dots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\dfrac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

The mean between the two results of the above Rogers-Ramanujan continued fractions is 0.97798855285, value very near to the ψ Regge slope 0.979:

Ψ	3	$m_c = 1500$	0.979	-0.09

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.99040073270864402755097375571330141546073279617855555551684...

0.99040073.... result very near to the dilaton value **0.989117352243** = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

From

AdS Vacua from Dilaton Tadpoles and Form Fluxes - *J. Mourad and A. Sagnotti* - arXiv:1612.08566v2 [hep-th] 22 Feb 2017 - March 27, 2018

We have:

$$\begin{aligned} e^{2C} &= \frac{2\,\xi\,e^{\frac{\phi}{2}}}{1\,\pm\,\sqrt{1\,-\,\frac{\xi T}{3}\,e^{2\phi}}} \\ \frac{h^2}{32} &= \frac{\xi^7\,e^{4\phi}}{\left(1\,\pm\,\sqrt{1\,-\,\frac{\xi T}{3}\,e^{2\phi}}\right)^7}\left[\frac{42}{\xi}\left(1\,\pm\,\sqrt{1\,-\,\frac{\xi\,T}{3}\,e^{2\phi}}\right)\,+\,5\,T\,e^{2\phi}\right]\,. \qquad (2.7) \end{aligned}$$

For

$T = \frac{16}{\pi^2}$

$\xi = 1$

we obtain:

(2*e^(0.989117352243/2)) / (1+sqrt(((1-1/3*16/(Pi)^2*e^(2*0.989117352243)))))

Input interpretation:

$$\frac{2\,e^{0.989117352243/2}}{1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\,e^{2\times0.989117352243}}}$$

Result:

0.83941881822... −
1.4311851867... i

Polar coordinates:

$r = 1.65919106525$ (radius), $\theta = -59.607521917°$ (angle)

1.65919106525….. result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Series representations:

$$\frac{2\, e^{0.98911735224300000/2}}{1+\sqrt{1-\frac{16\, e^{2\times 0.98911735224300000}}{3\pi^2}}} = \frac{2\, e^{0.49455867612150000}}{1+\sqrt{-\frac{16\, e^{1.97823470448600000}}{3\pi^2}}\ \sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.97823470448600000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{2\, e^{0.98911735224300000/2}}{1+\sqrt{1-\frac{16\, e^{2\times 0.98911735224300000}}{3\pi^2}}} = \frac{2\, e^{0.49455867612150000}}{1+\sqrt{-\frac{16\, e^{1.97823470448600000}}{3\pi^2}}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.97823470448600000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{2\, e^{0.98911735224300000/2}}{1+\sqrt{1-\frac{16\, e^{2\times 0.98911735224300000}}{3\pi^2}}} = \frac{2\, e^{0.49455867612150000}}{1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\, e^{1.97823470448600000}}{3\pi^2}-z_0\right)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From

$$\frac{h^2}{32} = \frac{\xi^7 e^{4\phi}}{\left(1 \pm \sqrt{1-\frac{\xi T}{3}e^{2\phi}}\right)^7}\left[\frac{42}{\xi}\left(1 \pm \sqrt{1-\frac{\xi T}{3}e^{2\phi}}\right) + 5\, T e^{2\phi}\right]$$

we obtain:

e^(4*0.989117352243) / (((1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243)))))^7 [42(1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243)))+5*16/(Pi)^2*e^(2*0.989117352243)]

Input interpretation:

$$\frac{e^{4\times 0.989117352243}}{\left(1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}}\right)+5\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}\right)$$

Result:

$50.84107889\ldots -$
$20.34506335\ldots i$

Polar coordinates:

$r = 54.76072411$ (radius), $\theta = -21.80979492°$ (angle)

54.76072411…..

Series representations:

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.9891173522430000}}{\pi^2}\right)e^{4\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7 =$$

$$\left(2\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)\Big/\left(\pi^2\left(1+\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)$$

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.9891173522430000}}{\pi^2}\right)e^{4\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7 =$$

$$\left(2\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/$$

$$\left(\pi^2\left(1+\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)$$

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.9891173522430000}}{\pi^2}\right)e^{4\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7 =$$

$$\left(2\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}-z_0\right)^k z_0^{-k}}{k!}\right)\right)\Big/$$

$$\left(\pi^2\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)$$

for $(\text{not }(z_0\in\mathbb{R}\text{ and }-\infty<z_0\leq 0))$

From which:

e^(4*0.989117352243) / (((1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243))))))^7 [42(1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243)))+5*16/(Pi)^2*e^(2*0.989117352243)]*1/34

Input interpretation:

$$\frac{e^{4\times 0.989117352243}}{\left(1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}}\right)^7}$$
$$\left(42\left(1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}}\right)+5\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}\right)\times\frac{1}{34}$$

Result:

1.495325850… −
0.5983842161… i

Polar coordinates:

$r = 1.610609533$ (radius), $\theta = -21.80979492°$ (angle)

1.610609533…. result that is a good approximation to the value of the golden ratio 1.618033988749...

Series representations:

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.98911735224300000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.98911735224300000}}{\pi^2}\right)e^{4\times 0.98911735224300000}\right)\Big/\left(34\left(1+\sqrt{1-\frac{16\,e^{2\times 0.98911735224300000}}{3\,\pi^2}}\right)^7\right)=$$
$$\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)\Big/$$
$$\left(17\,\pi^2\left(1+\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)$$

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.9891173522430000}}{\pi^2}\right)e^{4\times 0.9891173522430000}\right)\Big/\left(34\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7\right)=$$

$$\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/$$

$$\left(17\,\pi^2\left(1+\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)$$

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.9891173522430000}}{\pi^2}\right)e^{4\times 0.9891173522430000}\right)\Big/\left(34\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7\right)=$$

$$\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$

$$\left(17\,\pi^2\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Now, we have:

$$e^{2C} \;=\; \frac{2\,\xi\, e^{-\frac{\phi}{2}}}{1 \;+\; \sqrt{1 \;+\; \frac{\xi\Lambda}{3}\, e^{2\phi}}}\;, \tag{2.9}$$

$$\frac{h^2}{32} \;=\; \frac{e^{-4\phi}}{\left[1 \;+\; \sqrt{1 \;+\; \frac{\Lambda}{3}\, e^{2\phi}}\right]^7}\;\left[42\left(1 \;+\; \sqrt{1 \;+\; \frac{\Lambda}{3}\, e^{2\phi}}\right) \;-\; 13\,\Lambda\, e^{2\phi}\right]\;. \tag{2.10}$$

For:

$\xi = 1$

$\Lambda \simeq \frac{4\pi^2}{25}$

$\phi = 0.989117352243$

From

$$e^{2C} \;=\; \frac{2\,\xi\, e^{-\frac{\phi}{2}}}{1 \;+\; \sqrt{1 \;+\; \frac{\xi\Lambda}{3}\, e^{2\phi}}}\;,$$

we obtain:

((2*e^(-0.989117352243/2))) / ((((1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))))))

Input interpretation:

$$\frac{2\, e^{-0.989117352243/2}}{1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^2\right)\right) e^{2\times 0.989117352243}}}$$

Result:

0.382082347529...

0.382082347529….

Series representations:

$$\frac{2\,e^{-0.9891173522430000/2}}{1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}} = 2\Bigg/\left(e^{0.4945586761215000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$\frac{2\,e^{-0.9891173522430000/2}}{1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}} = 2\Bigg/\left(e^{0.4945586761215000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$\frac{2\,e^{-0.9891173522430000/2}}{1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}} = \frac{2}{e^{0.4945586761215000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

From which:

1+1/(((4((2*e^(-0.989117352243/2))) / ((((1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))))))))

Input interpretation:

$$1+\frac{1}{4\times\frac{2\,e^{-0.989117352243/2}}{1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}}}$$

Result:

1.65430921270...

1.6543092….. We note that, the result 1.6543092... is very near to the 14th root of the following Ramanujan’s class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164.2696$ i.e. 1.65578...

Indeed:

$$G_{505} = P^{-1/4}Q^{1/6} = (\sqrt{5}+2)^{1/2}\left(\frac{\sqrt{5}+1}{2}\right)^{1/4}(\sqrt{101}+10)^{1/4}$$
$$\times\left((130\sqrt{5}+29\sqrt{101})+\sqrt{169440+7540\sqrt{505}}\right)^{1/6}.$$

Thus, it remains to show that

$$(130\sqrt{5}+29\sqrt{101})+\sqrt{169440+7540\sqrt{505}} = \left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3,$$

which is straightforward. □

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1{,}65578\ldots$$

Series representations:

$$1+\frac{1}{\frac{4\left(2e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{(4\pi^2)e^{2\times0.98911735224300000}}{3\times25}}}} =$$

$$1+\frac{e^{0.4945586761215000}}{8}+\frac{1}{8}e^{0.4945586761215000}\sqrt{\frac{4e^{1.978234704486000}\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}$$

$$1+\frac{1}{\frac{4\left(2\,e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.98911735224300000}}{3\times 25}}}}=$$

$$1+\frac{e^{0.49455867612150000}}{8}+\frac{1}{8}\,e^{0.49455867612150000}\sqrt{\frac{4\,e^{1.97823470448600000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.97823470448600000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

$$1+\frac{1}{\frac{4\left(2\,e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.98911735224300000}}{3\times 25}}}}=1+\frac{e^{0.49455867612150000}}{8}+$$

$$\frac{1}{8}\,e^{0.49455867612150000}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.97823470448600000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}$$

$$\text{for (not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\leq 0))$$

And from

$$\frac{h^2}{32}=\frac{e^{-4\phi}}{\left[1+\sqrt{1+\frac{\Lambda}{3}\,e^{2\phi}}\right]^7}\left[42\left(1+\sqrt{1+\frac{\Lambda}{3}\,e^{2\phi}}\right)-13\,\Lambda\,e^{2\phi}\right].$$

we obtain:

e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243)))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243)))-13*(4Pi^2)/25*e^(2*0.989117352243)]

Input interpretation:

$$\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}\right)-13\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}\right)$$

Result:

$-0.034547055658\ldots$

-0.034547055658…

Series representations:

$$\left(\left(42\left(1+\sqrt{1+\frac{\left(4\pi^2\right)e^{2\times 0.98911735224300000}}{3\times 25}}\right)-\frac{1}{25}\left(4\pi^2\right)13\,e^{2\times 0.98911735224300000}\right)e^{-4\times 0.98911735224300000}\right)\Bigg/\left(1+\sqrt{1+\frac{\left(4\pi^2\right)e^{2\times 0.98911735224300000}}{3\times 25}}\right)^7 =$$

$$-\left(\left(42\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)\Bigg/\left(25\,e^{5.934704113458000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)\right)$$

$$\left(\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)e^{-4\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7 =$$

$$-\left(\left(42\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/\left(25\,e^{5.934704113458000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)\right)$$

$$\left(\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)e^{-4\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7 =$$

$$-\left(\left(42\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)\right)\Big/\left(25\,e^{5.934704113458000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From which:

47 *1/(((-1/((((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-13*(4Pi^2)/25*e^(2*0.989117352243))]))))))))

Input interpretation:

$$47\left(-\left(1/1/\left(\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}\right)\right)\right)\right)\right)$$

Result:

1.6237116159...

1.6237116159…. result that is an approximation to the value of the golden ratio 1.618033988749...

Series representations:

$$-\left(47/1/\left(e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{\left(4\pi^2\right)e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}\left(4\pi^2\right)13\,e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{\left(4\pi^2\right)e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)=$$

$$\left(1974\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)\Big/\left(25\,e^{5.934704113458000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)$$

$$-\Bigg(47 \Big/ 1 \Big/ \Bigg(e^{-4\times 0.9891173522430000}\Bigg(42\Bigg(1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}\,(4\,\pi^2)\,13\,e^{2\times 0.9891173522430000}\Bigg)\Bigg)\Bigg)\Bigg)\Big/ \left(1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\Bigg)=$$

$$\Bigg(1974\Bigg(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\,\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\Bigg)\Bigg)\Big/\Bigg(25\,e^{5.934704113458000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\Bigg)$$

$$-\Bigg(47 \Big/ 1 \Big/ \Bigg(e^{-4\times 0.9891173522430000}\Bigg(42\Bigg(1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}\,(4\,\pi^2)\,13\,e^{2\times 0.9891173522430000}\Bigg)\Bigg)\Bigg)\Bigg)\Big/ \left(1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\Bigg)=$$

$$\Bigg(1974\Bigg(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\,\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\Bigg)\Bigg)\Big/\Bigg(25\,e^{5.934704113458000}\left(1+\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^7\Bigg)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

And again:

32((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-13*(4Pi^2)/25*e^(2*0.989117352243))]))))

Input interpretation:

$$32\left(\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^{2}\right)\right)e^{2\times 0.989117352243}}\right)^{7}}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^{2}\right)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}\left(4\pi^{2}\right)\right)e^{2\times 0.989117352243}\right)\right)\right)$$

Result:

-1.1055057810...

-1.1055057810….

We note that the result -1.1055057810…. is very near to the value of Cosmological Constant, less 10^{-52} , thence 1.1056, with minus sign

Series representations:

$$\left(32\, e^{-4\times 0.9891173522430000} \left(42 \left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\; - \frac{1}{25}\,(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right) \Big/ \left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7 =$$

$$-\Bigg(\Bigg(1344 \Bigg(-25\, e^{1.978234704486000} + 52\, e^{3.956469408972000}\, \pi^2 - 25\, e^{1.978234704486000} \sqrt{\frac{4\, e^{1.978234704486000}\, \pi^2}{75}} \sum_{k=0}^{\infty} \left(\frac{75}{4}\right)^k \left(e^{1.978234704486000}\, \pi^2\right)^{-k} \binom{\frac{1}{2}}{k}\Bigg)\Bigg) \Big/ \Bigg(25\, e^{5.934704113458000} \left(1+\sqrt{\frac{4\, e^{1.978234704486000}\, \pi^2}{75}} \sum_{k=0}^{\infty} \left(\frac{75}{4}\right)^k \left(e^{1.978234704486000}\, \pi^2\right)^{-k} \binom{\frac{1}{2}}{k}\right)^7\Bigg)\Bigg)$$

$$\left(32\, e^{-4\times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}} - \frac{1}{25}\, (4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right) \Bigg/ \left(1 + \sqrt{1 + \frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7 = -\left(\left(1344 \left(-25\, e^{1.978234704486000} + 52\, e^{3.956469408972000}\, \pi^2 - 25\, e^{1.978234704486000} \sqrt{\frac{4\, e^{1.978234704486000}\, \pi^2}{75}} \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k \left(e^{1.978234704486000}\, \pi^2\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)\right) \Bigg/ \left(25\, e^{5.934704113458000} \left(1 + \sqrt{\frac{4\, e^{1.978234704486000}\, \pi^2}{75}} \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k \left(e^{1.978234704486000}\, \pi^2\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)\right)$$

$$\left(32\, e^{-4\times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}} - \frac{1}{25}\, (4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right) \Bigg/ \left(1 + \sqrt{1 + \frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7 = -\left(\left(1344 \left(-25\, e^{1.978234704486000} + 52\, e^{3.956469408972000}\, \pi^2 - 25\, e^{1.978234704486000} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4\, e^{1.978234704486000}\, \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!}\right)\right) \Bigg/ \left(25\, e^{5.934704113458000} \left(1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4\, e^{1.978234704486000}\, \pi^2}{75} - z_0\right)^k z_0^{-k}}{k!}\right)^7\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

And:

-[32((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-13*(4Pi^2)/25*e^(2*0.989117352243))]))))]^5

Input interpretation:

$$-\left(32\left(\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}\right)\right)\right)\right)^5$$

Result:

1.651220569...

1.651220569…. result very near to the 14th root of the following Ramanujan’s class invariant $Q=\left(G_{505}/G_{101/5}\right)^3=1164.2696$ i.e. 1.65578...

Series representations:

$$-\left(\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)^5 =$$

$$\left(4\,385\,270\,057\,140\,224\left(-25+52\, e^{1.978234704486000}\,\pi^2-25\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^5\right)\Big/$$

$$\left(9\,765\,625\, e^{19.78234704486000}\left(1+\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^{35}\right)$$

$$-\left(\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\,\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}\,(4\,\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\,\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)^5 =$$

$$\left(4\,385\,270\,057\,140\,224\left(-25+52\, e^{1.978234704486000}\,\pi^2-25\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^5\right)\Big/$$

$$\left(9\,765\,625\, e^{19.78234704486000}\left(1+\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{35}\right)$$

$$-\left(\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\,\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}\,(4\,\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\,\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)^5 =$$

$$\left(4\,385\,270\,057\,140\,224\left(-25+52\, e^{1.978234704486000}\,\pi^2-25\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\, e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^5\right)\Big/$$

$$\left(9\,765\,625\, e^{19.78234704486000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\, e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^{35}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

We obtain also:

-[32((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-13*(4Pi^2)/25*e^(2*0.989117352243))])))))]^1/2

Input interpretation:

$$-\sqrt{\left(32\left(\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^{2}\right)\right)e^{2\times 0.989117352243}}\right)^{7}}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^{2}\right)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}\left(4\pi^{2}\right)\right)e^{2\times 0.989117352243}\right)\right)\right)\right)}$$

Result:

-0

$1.0514303501\ldots\, i$

Polar coordinates:

$r = 1.05143035007$ (radius), $\theta = -90°$ (angle)

1.05143035007

Series representations:

$$-\sqrt{\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)}=-\frac{8}{5}\sqrt{21}$$

$$\sqrt{\left(\left(25-52\, e^{1.978234704486000}\,\pi^2+25\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\Big/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)\right)}$$

$$-\sqrt{\left(\left(32\,e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)}=-\frac{8}{5}\sqrt{21}\sqrt{\left(\left(25-52\,e^{1.978234704486000}\,\pi^2+25\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)\right)}$$

$$-\sqrt{\left(\left(32\,e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)}=$$
$$-\frac{8}{5}\sqrt{21}\sqrt{\left(\left(25-52\,e^{1.978234704486000}\,\pi^2+25\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/\left(e^{3.956469408972000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)\right)}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

1 / -[32((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-13*(4Pi^2)/25*e^(2*0.989117352243))])))))]^1/2

Input interpretation:

$$-\left(1\middle/\left(\sqrt{\left(32\left(\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}\right)-13\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}\right)\right)\right)}\right)\right)$$

Result:

$0.95108534763\ldots i$

Polar coordinates:

$r = 0.95108534763$ (radius), $\theta = 90°$ (angle)

0.95108534763

We know that the primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index $n_s = 0.965 \pm 0.004$, consistent with the predictions of slow-roll, single-field, inflation.

Thence 0.95108534763 is a result very near to the spectral index n_s , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}}}\approx 0.9568666373$$

Series representations:

$$-\left(1\middle/\left(\sqrt{\left(\left(32\,e^{-4\times0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times0.9891173522430000}}{3\times25}}\right)-\frac{1}{25}(4\pi^2)\,13\,e^{2\times0.9891173522430000}\right)\right)\middle/\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times0.9891173522430000}}{3\times25}}\right)^7\right)}\right)\right)=$$

$$-\left(5\middle/\left(8\sqrt{21}\sqrt{\left(\left(25-52\,e^{1.978234704486000}\,\pi^2+25\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\middle/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)\right)}\right)\right)$$

$$-\left(1\Big/\left(\sqrt{\left(\left(32\,e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)-\frac{1}{25}\,(4\,\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)}\right)\right)=$$

$$-\left(5\Big/\left(8\,\sqrt{21}\,\sqrt{\left(\left(25-52\,e^{1.978234704486000}\,\pi^2+25\,\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)\right)}\right)\right)$$

$$-\left(1\Big/\left(\sqrt{\left(\left(32\,e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)}\right)\right)=$$

$$-\left(5\Big/\left(8\sqrt{21}\sqrt{\left(\left(25-52\,e^{1.978234704486000}\,\pi^2+25\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/\left(e^{3.956469408972000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)\right)}\right)\right)$$

$\text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

From the previous expression

$$\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}(4\pi^2)\right)e^{2\times 0.989117352243}}\right)^7\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}(4\pi^2)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}(4\pi^2)\right)e^{2\times 0.989117352243}\right)\right)}$$

= -0.034547055658…

we have also:

1+1/(((4((2*e^(-0.989117352243/2)) / ((((1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243)))))))))) + (-0.034547055658)

Input interpretation:

$$1+\frac{1}{4\times\frac{2\,e^{-0.989117352243/2}}{1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^2\right)\right)e^{2\times0.989117352243}}}}-0.034547055658$$

Result:

1.61976215705...

1.61976215705….. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Series representations:

$$1+\frac{1}{\frac{4\left(2\,e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{\left(4\,\pi^2\right)e^{2\times0.98911735224300000}}{3\times25}}}}-0.0345470556580000=$$

$$0.9654529443420000+\frac{e^{0.4945586761215000}}{8}+\frac{1}{8}\,e^{0.4945586761215000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}$$

$$1+\frac{1}{\frac{4\left(2\,e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{\left(4\,\pi^2\right)e^{2\times0.98911735224300000}}{3\times25}}}}-0.0345470556580000=$$

$$0.9654529443420000+\frac{e^{0.4945586761215000}}{8}+\frac{1}{8}\,e^{0.4945586761215000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

$$1+\frac{1}{\frac{4\left(2\,e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.98911735224300000}}{3\times 25}}}}-0.0345470556580000=$$

$$0.9654529443420000+\frac{e^{0.4945586761215000}}{8}+$$

$$\frac{1}{8}\,e^{0.4945586761215000}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}$$

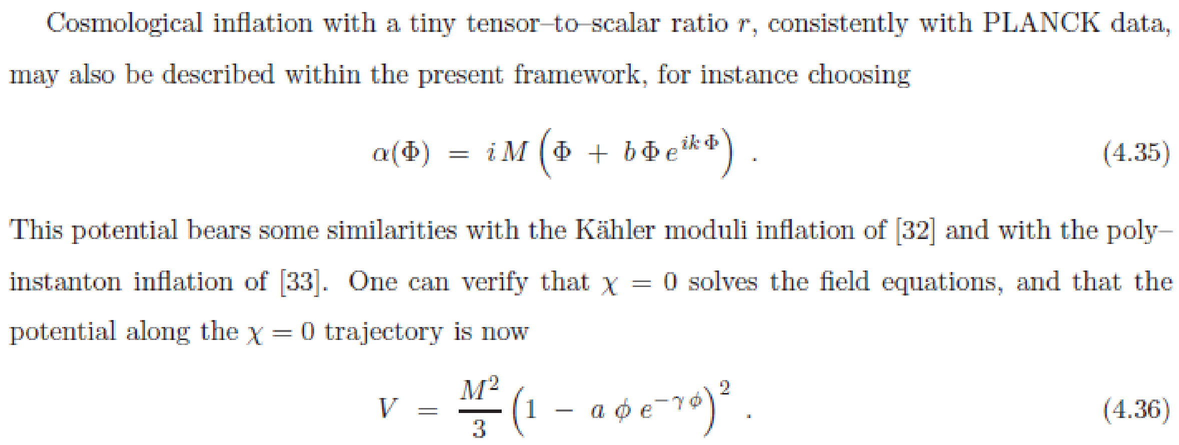
for (not $(z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0))$

From

Properties of Nilpotent Supergravity

E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:



Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi)\;=\;i\,M\left(\Phi\;+\;b\,\Phi\,e^{ik\,\Phi}\right)\;. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi\,=\,0$ solves the field equations, and that the potential along the $\chi=0$ trajectory is now

$$V\;=\;\frac{M^2}{3}\left(1\;-\;a\;\phi\;e^{-\gamma\,\phi}\right)^2\;. \tag{4.36}$$

We analyzing the following equation:

$$V\;=\;\frac{M^2}{3}\left(1\;-\;a\;\phi\;e^{-\gamma\,\phi}\right)^2\;.$$

$$\phi\;=\;\varphi\;-\;\frac{\sqrt{6}}{k}\;,$$

$$a = \frac{b\gamma}{e} < 0\,, \qquad \gamma = \frac{k}{\sqrt{6}} < 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V = \frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V = \frac{1}{3}(0.0814845\,b+1)^2 M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right) = 0.054323\,(0.0814845\,b + 1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M, V)}{\partial V} = \frac{154317775011120075}{36961748\,(226802245 + 18480874\,b)\,M^2}$$

$$\frac{\partial b(M, V)}{\partial M} = -\frac{\frac{226802245}{18480874} + b}{M}$$

$$\frac{\partial M(b, V)}{\partial V} = \frac{154317775011120075}{2\,(226802245 + 18480874\,b)^2\,M}$$

$$\frac{\partial M(b, V)}{\partial b} = -\frac{18480874\,M}{226802245 + 18480874\,b}$$

$$\frac{\partial V(b, M)}{\partial M} = \frac{2\,(226802245 + 18480874\,b)^2\,M}{154317775011120075}$$

$$\frac{\partial V(b, M)}{\partial b} = \frac{36961748\,(226802245 + 18480874\,b)\,M^2}{154317775011120075}$$

Global minimum:

$$\min\left\{\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right\} = 0 \text{ at } (b, M) = (-16, 0)$$

Global minima:

$$\min\left\{\frac{1}{3} M^2 \left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $b=-\frac{226\,802\,245}{18\,480\,874}$

$$\min\left\{\frac{1}{3} M^2 \left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\, M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M\neq 0)$$

we obtain

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

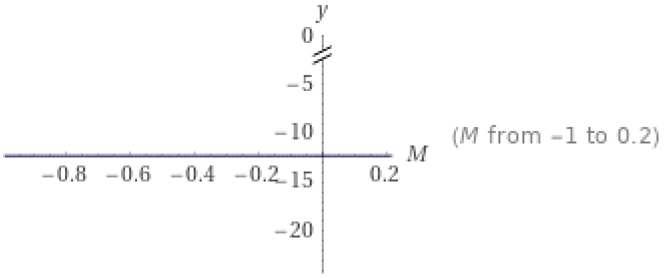
Input interpretation:

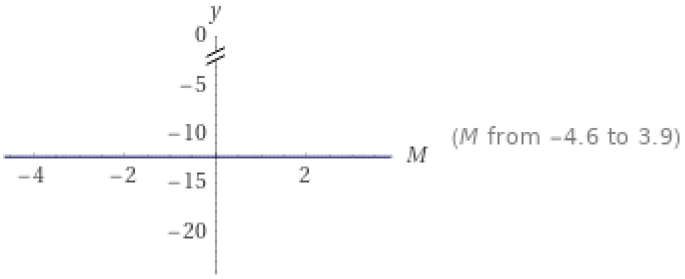
$$\frac{225.913\left(-0.054323\, M^2 + 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

Result:

$$\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}-0.054323\, M^2\right)}{M^2}$$

Plots:





Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2 - 1.21228\times 10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times 10^{-7}\sqrt{M^4} - 12.2723\, M^2}{M^2}$$

Expanded form:

$$\frac{1.48774\times 10^{-7}\sqrt{M^4}}{M^2}-12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774\times 10^{-7}\sqrt{M^4}}{M^2}-12.2723\right)+O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

$$-12.2723$$

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right)=\frac{3.55271\times 10^{-15}}{M}$$

Indefinite integral:

$$\int\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\,dM=$$

$$\frac{1.48774\times 10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}=-12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM=0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V = \frac{1}{3}(0.0814845\,b + 1)^2 M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

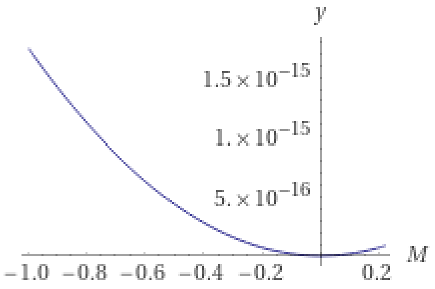
Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

Result:

0

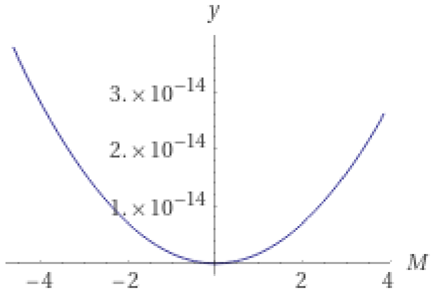
Plots: **(possible mathematical connection with an open string)**



(M from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(M from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$O(M^{62194})$

(Taylor series)

Series expansion at M = ∞:

$1.75541\times10^{-15}\, M^2 + O\left(\left(\frac{1}{M}\right)^{62194}\right)$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^\infty \left(\frac{1}{3} M^2 \left(1 + \frac{18.4084 \left(-0.054323\, M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} \right)^2 - 1.75541 \times 10^{-15}\, M^2 \right) dM = 0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3} \left(0.0814845 \times \frac{225.913 \left(-0.054323\, M^2 + 6.58545 \times 10^{-10} \sqrt{M^4} \right)}{M^2} + 1 \right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3} \left(0.0814845 \times \frac{225.913 \left(-0.054323\, (-0.5)^2 + 6.58545 \times 10^{-10} \sqrt{(-0.5)^4} \right)}{(-0.5)^2} + 1 \right)^2 \left(-0.5^2 \right)$$

Result:

$$-4.38851344947464545348970783378088020833333333333333333333\ldots \times 10^{-16}$$

$-4.38851344947 * 10^{-16}$

For M = 0.2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^{2}+6.58545\times10^{-10}\sqrt{M^{4}}\right)}{M^{2}}+1\right)^{2}M^{2}$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times0.2^{2}+6.58545\times10^{-10}\sqrt{0.2^{4}}\right)}{0.2^{2}}+1\right)^{2}\times0.2^{2}$$

Result:

$$7.0216215191594327255835325340494083333333333333333333333333\ldots\times10^{-17}$$

$7.021621519159*10^{-17}$

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^{2}+6.58545\times10^{-10}\sqrt{M^{4}}\right)}{M^{2}}+1\right)^{2}M^{2}$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 3^2+6.58545\times 10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times 3^2$$

Result:

$$1.579864841810872363256294820161116875\times 10^{-14}$$

$1.57986484181*10^{-14}$

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 2^2+6.58545\times 10^{-10}\sqrt{2^4}\right)}{2^2}+1\right)^2\times 2^2$$

Result:

$$7.0216215191594327255835325340494083333333333333333333333333\ldots \times 10^{-15}$$

7.021621519*10^{-15}

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$$5.9776991059\ldots \times 10^{-8}$$

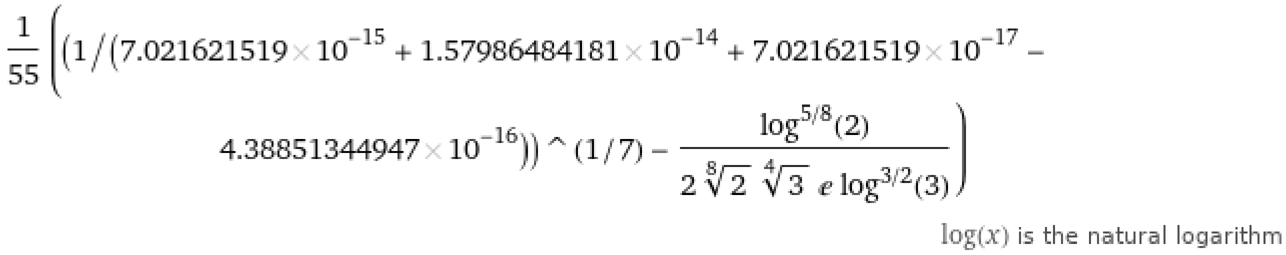
5.9776991059*10^{-8} result very near to the Planck's electric flow 5.975498 × 10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}}\, l_{\mathrm{P}}^{2} = \phi_{\mathrm{P}}\, l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*(([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)])))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:



$$\frac{1}{55}\left((1/(7.021621519\times10^{-15}+1.57986484181\times10^{-14}+7.021621519\times10^{-17}-4.38851344947\times10^{-16}))\,\hat{}\,(1/7)-\frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

log(*x*) is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_P = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_P = \frac{F_P}{q_P} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck’s Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^{2} = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

$5.975498*10^{-8}$ V*m Lorentz-Heaviside value

Planck’s Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

$1.042940*10^{27}$ V Lorentz-Heaviside value

Relationship between Planck’s Electric Flux and Planck’s Electric Potential

E_P * l_P = $(1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306\times 10^{61})\times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

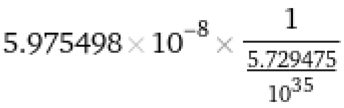
$1.042939771935\times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

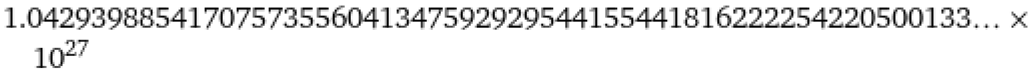
$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:



$$5.975498\times10^{-8}\times\frac{1}{\frac{5.729475}{10^{35}}}$$

Result:



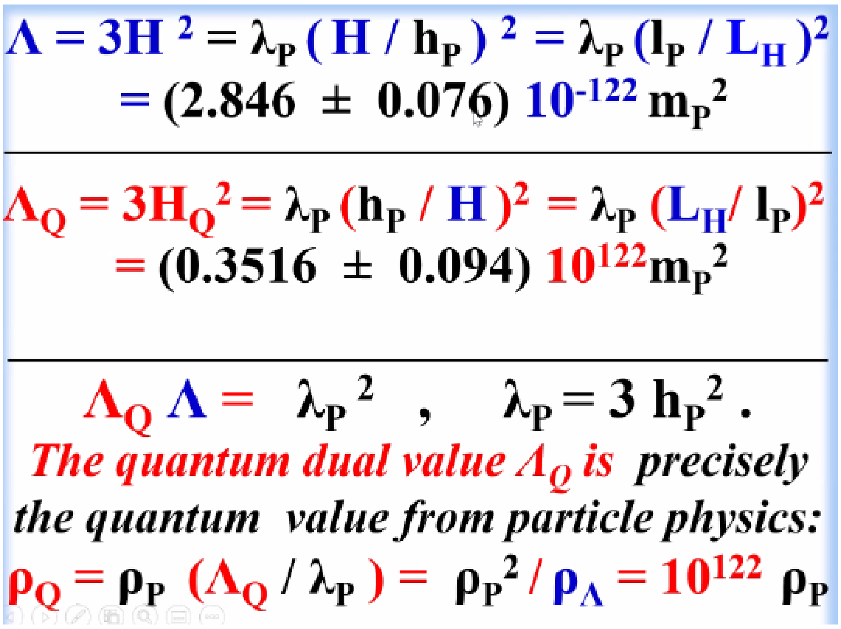
$1.04293988541707573556041347592929544155441816222254220500133\ldots\times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

From:

Planckian and Trans-Planckian physics in black holes and quantum space-time - *Norma Sanchez* (CNRS OP-SU Paris & Chalonge-de Vega School) - Talk - 02.12.2021

Scheme



$$\Lambda = 3H^2 = \lambda_P (H / h_P)^2 = \lambda_P (l_P / L_H)^2$$
$$= (2.846 \pm 0.076)\ 10^{-122}\ m_P^2$$

$$\Lambda_Q = 3H_Q^2 = \lambda_P (h_P / H)^2 = \lambda_P (L_H / l_P)^2$$
$$= (0.3516 \pm 0.094)\ 10^{122} m_P^2$$

$$\Lambda_Q\, \Lambda = \lambda_P^2 \quad , \quad \lambda_P = 3\, h_P^2 .$$

The quantum dual value Λ_Q is precisely the quantum value from particle physics:

$$\rho_Q = \rho_P\ (\Lambda_Q / \lambda_P) = \rho_P^2 / \rho_\Lambda = 10^{122}\ \rho_P$$

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